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An Approach for a Decision Support Systems to optimize Water Tanks in Water Supply Systems by combining Network Reduction, Optimization and Simulation

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Abstract: Since the last two decades, the water consumption in Germany is decreasing, which causes the water tanks and pipes in a water supply system to work inefficiently. This paper proposes an approach for a decision support system, which helps to decide how to plan new water tanks and resize existing tanks in water supply systems. The approach uses a combination of network reduction, mathematical optimization and hydraulic simulation. The mathematical optimization model is a nonconvex Mixed Integer Quadratically Constrained Program (MIQCP), which is solved by a piecewise linearization. As this may lead to many binary variables and therefore high computational times, the size of the water supply system model is reduced before building the optimization model. By applying several network reduction techniques there may occur some hydraulic differences between the original network model and the reduced network model. To make sure that the solution obtained in the optimization process is feasible in the original water supply system, the solution is verified by a hydraulic simulation tool.

Keywords: Decision support system, Water tanks, Mathematical optimization, Nonconvex MIQCP, Network reduction, Hydraulic simulation

Konzept für ein Entscheidungsunterstützungssystem für die Optimierung von Wasserbehältern in einem Wasserversorgungssystem – Kombination von Netzwerkreduktion, Optimierung und Simulation

Abstract [German]: Der Wassergebrauch in Deutschland ist in den letzten zwei Jahrzehnten zurückgegangen, was dazu führt, dass manche Wasserbehälter und Wasserrohre in den Wasserversorgungssystemen ineffizient arbeiten. Dieser Artikel präsentiert ein Konzept für ein Entscheidungsunterstützungssystem, das dabei helfen soll, neue Wasserbehälter zu planen und zu dimensionieren. Weiterhin kann für existierende Wasserbehälter entschieden werden, ob ihre Dimension verändert werden sollte. Das Konzept kombiniert Netzreduktionstechniken, mathematische Optimierungsmethoden sowie hydraulische Simulationsmethoden. Das mathematische Optimierungsmodell ist ein nichtkonvexes Mixed Integer Quadratically Constrained Program (MIQCP), das mit Hilfe einer stückweisen Linearisierung gelöst wird. Da dies zu einer großen Anzahl an Binärvariablen und damit zu hohen Lösungszeiten führen kann, wird die Größe des Modells des Wasserversorgungsnetzes reduziert, bevor das Optimierungsmodell aufgestellt wird. Durch die Anwendung verschiedener Netzwerkreduktionstechniken kann es passieren, dass Unterschiede zwischen der Hydraulik in dem reduzierten Modell und der Hydraulik im Originalmodell auftreten. Um sicherzustellen, dass eine Lösung, die durch die Optimierung gewonnen wurde, auch zulässig ist, wenn sie im Originalmodell betrachtet wird, wird die Lösung durch ein hydraulisches Simulationstool validiert.

Keywords [German]: Entscheidungsunterstützungssystem, Wasserbehälter, Mathematische Optimierung, Nichtkonvexes MICQP, Netzwerkreduktion, Hydraulische Simulation

1 Introduction

In Germany, the municipal utilities are responsible for the water supply. In recent years they are facing an increasing cost pressure (Spindler 2012), which is caused by different reasons. Increasing energy prices are yielding increasing energy costs for the utilities. Aside from that, due to the discussions about the liberalization of the water market the utilities have to prepare themselves for competing against other utilities (Brackemann et al. 2000, p. 41). Another important reason is the decreasing water consumption in Germany in the last two decades. This is important, because of the following effect: When the water supply systems were designed the planners assumed an increasing demand of water in the future. But due to water saving measures the water consumption decreased, and therefore a lot of components in water supply systems do not have the right dimensions to work efficiently. This is mainly true for pipes and water tanks.

Decision support systems may help to decrease the costs for the utilities and to increase the efficiency of the components. Such systems have to handle the complexity of a water supply system, which is caused by the size of the network and the number of different components and their specific properties. In addition, the hydraulic properties are difficult to handle because they are considered to be nonlinear when stated in mathematical terms. A decision support system may include mathematical optimization as well as hydraulic simulation and should propose a solution in reasonable time.

Mathematical optimization in water supply systems is a well-known problem in the literature. Most of the research tackles the problem of pipe design in a water distribution system. Alperovits and Shamir (1977) present an optimization model, which minimizes the costs of the pipe design for a given network. It contains of not only sizing the components but setting the operational decisions for pumps and valves. The model is solved via a linear programming gradient method that is based on a hierarchical decomposition of the optimization problem. Based on the model of Alperovits and Shamir (1977) and some extensions (Kessler and Shamir 1989), Eiger et al. (1994) present another approach to solve this problem. This approach solves the overall design problem "globally by a branch and bound algorithm using non-smooth optimization and duality theory" (Eiger et al. 1994, p. 2637). Other authors considering the pipe optimization problem solve it via genetic algorithms (Simpson et al. 1994; Dandy et al. 1996; Djebedjian et al. 2006), tabu search (Cunha and Ribeiro 2004) or evolutionary algorithms in combination with hydraulic simulation (Liong and Atiquzzaman 2004; Reehuis 2010). There are some works considering the optimization of water tanks. Schmidt and Plate (1983) focus on the optimization of the operation of water tanks that are used for irrigation purposes. Lansey and Mays (1989) present a more detailed model for designing a water distribution system. In this model the design of tanks and pumps is considered with respect to daily demand patterns. Vamvakeridou-Lyroudia (2007) present an optimization model that optimizes water tanks in the process of water distribution system design optimization. Those three models are solved via a combination of optimization and simulation in different manners. Schmidt and Plate (1983) present an approach that uses a Monte Carlo method in combination with a dynamic programming technique. Vamvakeridou-Lyroudia (2007) proposes an approach that combines extended time period simulation to determine inflow and outflow of water tanks and a genetic algorithm that chooses new levels for the tanks. Lansey and Mays (1989) solve the proposed model via a combination of a problem-reduction technique, nonlinear programming techniques and a hydraulic simulator. The problem-reduction technique is a variable-reduction technique called generalized reduced gradient (GRG) (Mantell and Lasdon 1977), which reduces the problem size. The proposed optimization model is partitioned into subsets, which are solved implicitly using a hydraulic simulator. Therefore, the size of the remaining optimization model is reduced and then, the model is easier to solve. There are other authors who consider reducing the size of the optimization model. Haehnlein (2008) developed an optimization model for pump control. The considered network is approximated by a simplified semi-virtual hydraulic model, which only takes into account the necessary components. The problem is then solved by discrete dynamic programming. Burgschweiger et al. (2009) deal with the optimization of the operative planning in water supply systems. Before applying a gradient-based optimization algorithm the size of the network is reduced by using a few network reduction techniques.

This paper focuses on an approach for a decision support system for planning water tanks. The overall goal is to minimize investment and operational costs of water tanks. Therewith the optimal location and dimension of the tanks is decided as well. The paper is organized as follows. Section 2 gives a short overview of a water supply system and its components. Section 3 presents a mathematical formulation of the optimization model for optimizing water tanks in water supply systems. The proposed model is a nonconvex Mixed Integer Quadratically Constrained Program (MIQCP) and is solved by using a piecewise linearization technique described in Section 4. The technique needs some binary variables, whose number depends on the number of pipes and the number of time steps considered in the model. As a water supply system may be very large, i.e. there may be a lot of pipes, the optimization model will become very large and will have a lot of binary variables. Hence, for real water supply systems it is often not possible to solve the optimization model in reasonable time. To overcome this effect, the size of the water supply system model is reduced before building the optimization model. The reduction techniques that are used are described in Section 5. These techniques may lead to a so called *hydraulic error,* which may affect the feasibility of an optimal solution that was determined during the optimization process. To verify the feasibility of the solution, it is transferred to a hydraulic simulation tool, which runs a simulation with the original sized network model, see Section 6. If the simulation tool detects infeasibilities, the locations of those infeasibilities are determined. At these locations the network reduction techniques are reversed so that in those parts of the network model the original structure is considered. This procedure is described in Section 6 as well. With the new network model the optimization model is built again and then solved. This process is done until the simulation tool verifies a solution. For this solution it cannot be guaranteed that it is globally optimal.

2 Overview of Water Supply Systems

Before stating the optimization model for optimizing water tanks the main components of a water supply systems are described.

A water supply system is built for the purpose to transport water from different sources to several clients. Clients may be public facilities, business companies or private households. In order to get an overview of a water supply system, the main components are displayed in Figure 1.

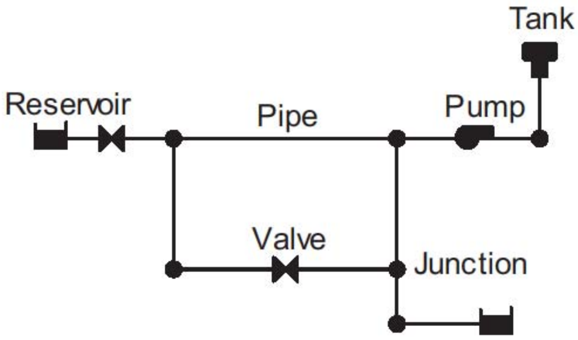

Fig. 1 Components in a water supply system

The components of a water supply system can be divided into nodes and links whose different meanings are described in the following.

A node can be a reservoir, which is a natural source of water such as a lake or a river. In the optimization model reservoirs are assumed to have an infinite amount of water that can be supplied into the system. A node may also represent a tank, which can store water or can supply water into the system. Every tank has specific properties such as a diameter, an initial, minimum and maximum level of water and therefore also an initial, minimum and maximum volume of water that can be stored in the tank. Nodes can also represent junctions where some links join together. At junctions water can enter or leave the system. All nodes have in common that they have a hydraulic head, which is mainly determined by the elevation of the node. If a node represents a tank, the hydraulic head is also affected by the water level of the tank. Reservoirs and tanks have an initial hydraulic head that influences the heads of all remaining nodes in the water supply system.

The hydraulic head is also influenced by the links and their properties. Most links represent pipes through which water flows from one node to another one. The main properties of a pipe are the diameter, the length and the roughness that depends on the material properties of the pipe (Mutschmann and Stimmelmayr 2007, pp. 588-589). Links can also be valves, which can control the pressure in a pipe or the amount of water flowing through a pipe. A link may also represent a pump, which transports water from nodes with a low elevation to nodes with a higher elevation. For this process a pump needs a specific amount of energy. For further details of the main components in water supply systems, we refer to Rossman (2000, pp. 27-46), Mutschmann and Stimmelmayr (2007) or Karger et al. (2008).

After introducing the main components of a water supply system, a hydraulic property in a water supply system can be stated: the head loss in a pipe. This head loss is determined by the hydraulic heads of start and end node of the pipe, the volumetric flow in the pipe, the diameter and length of the pipe as well as the roughness of the pipe. There are different formulas to describe the head loss in a pipe. The most important ones are the Darcy-Weisbach equation (Boulos et al. 2006, pp. 3-6 - 3-12), the Hazen-Williams equation (Boulos et al. 2006, pp. 3-12 - 3-14) and the Chezy-Manning equation (Boulos et al. 2006, pp. 3-14 - 3-16). In this paper we focus on the Darcy-Weisbach equation, which is stated below:

$$H_i - H_j = r_{ij} \cdot Q_{ij} \cdot |Q_{ij}|.$$

The parameters of this equation are described in Table 1 and can also be found in Karger et al. (2008, p. 223).

Tab. 1 Parameters of the Darcy-Weisbach equation

Name	Description
$\boldsymbol{H_i, H_j}$	Hydraulic head of node i, j, in m
$\boldsymbol{r_{ij}}$	Resistance coefficient in pipe ij, in h^2/m^5
$\boldsymbol{Q_{ij}}$	Volumetric flow rate in pipe ij, in m^3/h

Further details of the Darcy-Weisbach equation and the hydraulic properties are described in Karger (2008, pp. 223-225).

3 Optimization Model for Optimizing Water Tanks

Now, the optimization model is presented. A previous version of this model is described in Dohle and Suhl (2012). The optimization task is to minimize investment and operational costs of water tanks. Hence, it is decided if a tank should be built and if so, the optimal location and dimension are determined. This task is restricted by several constraints, such as satisfying the demand of water of all clients at any time or providing the necessary amount of water for fire-fighting during all time periods. In addition, there always has to be an amount of water in the system for any kind of incidence that can occur. Also, the physical properties of a water supply system have to be included, that are the volumetric flow through each pipe and the head at each node at any time step.

The mathematical formulation of this optimization problem is as follows. Let N define all nodes in the water supply system and let $B \subseteq N$ be the set of nodes at which a tank can be built or already exists. Then, $EB \subseteq B$ is the set of nodes where a tank already exists. The binary variable y_b indicates if tank b already exists or is built or not. Each tank has an initial amount of water $V0_b$, that is zero for non-existing tanks. The amount of water in a tank may vary between specific bounds, that is the minimal amount of water in a tank $Vmin_b$ and the maximal amount of water $Vmax_b$. Other parameters of each tank b are the investment costs $InvC_b$, the operational costs OpC_b and the diameter of the tank d_b. The amount of water which is stored in tank b at each time $t \in T$ is denoted by variable V_b^t. With that the hydraulic head L_b^t at each tank b can be calculated.

Each node n has a demand D_n^t, that can vary at different times t and can also have negative values, which represent an amount of water entering the system. The variable H_n^t denotes the head on each node n at any time t. To determine the heads in a water supply system, there is need of at least one reference head. In most cases this reference head is provided by an existing tank b and is defined by parameter $L0_b$. If there is no existing tank available, there is need of at least one reference node $r \in R \subseteq N$ which provides an initial head $InitH_r$.

The set $P \subseteq (N \times N)$ represents all pipes (i, j) in the system where i denotes the start node and j the end node of the pipe. Now, the variable Q_{ij}^t can be introduced. This variable stands for the volumetric flow rate through pipe (i, j) at time t. The volumetric flow rate is influenced by different properties of the pipe, such as the length of the pipe, the diameter of the pipe and the roughness of the pipe. All these parameters are integrated in the resistance coefficient r_{ij} of each pipe (i, j). In Table 2-4 the sets, parameters and variables of the optimization problem are summarized.

Tab. 2 Sets of the optimization model

Name	Description
$\boldsymbol{N}$	Set of nodes
$\boldsymbol{B} \subseteq \boldsymbol{N}$	Set of nodes where a tank can be built or already exists
$\boldsymbol{EB} \subseteq \boldsymbol{B}$	Set of nodes where a tank already exists
$\boldsymbol{R} \subseteq \boldsymbol{N}$	Set of reference nodes
$\boldsymbol{P} \subseteq (\boldsymbol{N} \times \boldsymbol{N})$	Set of pipes
$\boldsymbol{T}$	Set of time steps

Tab. 3 Parameters of the optimization model

Name	Description
$\boldsymbol{V0_b}$	Initial volume of water in tank b, in m^3
$\boldsymbol{Vmin_b}$	Minimal volume of water in tank b, in m^3
$\boldsymbol{Vmax_b}$	Maximal volume of water in tank b, in m^3
$\boldsymbol{L0_b}$	Initial hydraulic head of tank b, in m
$\boldsymbol{InvC_b}$	Investment costs of tank b, in MU
$\boldsymbol{OpC_b}$	Operational costs of tank b, in MU
$\boldsymbol{d_b}$	Diameter of tank b, in m
$\boldsymbol{D_n^t}$	Demand at node n at time t, in m^3
$\boldsymbol{InitH_r}$	Initial hydraulic head at reference node r, in m
$\boldsymbol{r_{ij}}$	Resistance coefficient of pipe (i,j), in h^2/m^5
$\boldsymbol{\Delta t}$	Length of one time step, in h
$\boldsymbol{M1, M2, M3, M4}$	Parameters to model some Big-M constraints

Tab. 4 Variables of the optimization model

Name	Description
$\boldsymbol{y_b}$	Binary variable, which indicates if tank b is built (1) or not (0)
$\boldsymbol{V_b^t}$	Amount of water stored in tank b at time t, in m^3/h
$\boldsymbol{L_b^t}$	Hydraulic head at tank b at time t, in m
$\boldsymbol{H_n^t}$	Hydraulic head at node n at time t, in m
$\boldsymbol{Q_{ij}^t}$	Volumetric flow rate between node i and j at time t, in m^3/h

Now, all sets, parameters and variables of the optimization model are introduced and the optimization model can be stated in mathematical terms, see (1)-(17).

$$
\begin{aligned}
&\min \sum_{b \in B} (InvC_b \cdot y_b \ + \ \max_{t \in T} OpC_b \cdot L_b^t) && && (1)\\
y_{eb} &= 1 && \forall\, eb \in EB && (2)\\
V_b^0 - V0_b &\leq M_1 \cdot (1 - y_b) && \forall\, b \in B && (3)\\
-V_b^0 + V0_b &\leq M_2 \cdot (1 - y_b) && \forall\, b \in B && (4)\\
V_b^t &\geq Vmin_b \cdot y_b && \forall\, b \in B, t \in T && (5)\\
V_b^t &\leq Vmax_b \cdot y_b && \forall\, b \in B, t \in T && (6)\\
L_b^t - \frac{4 \cdot V_b^{t-1} \cdot \Delta t}{\pi \cdot d_b^2} &\leq M_3 \cdot (1 - y_b) && \forall\, b \in B, t \in T && (7)\\
-L_b^t + \frac{4 \cdot V_b^{t-1} \cdot \Delta t}{\pi \cdot d_b^2} &\leq M_4 \cdot (1 - y_b) && \forall\, b \in B, t \in T && (8)\\
L_b^t &= H_b^t && \forall\, b \in B, t \in T && (9)\\
\sum_{(i,b) \in P} Q_{i,b}^t - \sum_{(b,j) \in P} Q_{b,j}^t &= V_n^t - V_n^{t-1} && \forall\, b \in B, t \in T && (10)\\
\sum_{(i,n) \in P} Q_{i,n}^t - \sum_{(n,j) \in P} Q_{n,j}^t &= D_n^t && \forall\, n \in N \backslash \{B\}, t \in T && (11)\\
H_r^1 &= InitH_r && \forall\, r \in R && (12)\\
H_i^t - H_j^t &= r_{i,j} \cdot Q_{i,j}^t \cdot |Q_{i,j}^t| && \forall\, (i,j) \in P, t \in T && (13)\\
V_b^t &\geq 0 && \forall\, b \in B, t \in T && (14)\\
y_b &\in \{0,1\} && \forall\, b \in B && (15)\\
H_n^t &\geq 0 && \forall\, n \in N, t \in T && (16)\\
Q_{i,j}^t &\ free && \forall (i,j) \in P, t \in T && (17)
\end{aligned}
$$

The objective function (1) minimizes the investment costs for building a new tank as well as the operational costs. As the water level indicates the height of the tank, this variable is used to determine the height of a tank that is at least needed for storing the water at all time steps. To identify this height, the maximal water level over all time steps is minimized. This variable is then multiplied by the operational costs. Constraint (2) represents the already existing tanks and sets the corresponding binary variable to one. The constraints (3)-(6) control the volume of water in the tanks. If a tank is built, an initial volume of this tank is set. Otherwise the initial volume of the tank is forced to be zero. In addition, the volume of water in an existing tank has to stay in its ranges. As the water level of a tank and the volume of water in a tank are related, the ranges of the water level are not controlled separately. The relationship of the water level and the volume of water in a tank are described in constraints (7) and (8). These constraints come into play only if a tank is built. Constraint (9) makes sure that the water level of the tank is related to the hydraulic head of the corresponding node. The mass conservation equations for each node with a tank and for each node without a tank are described in constraints (10) and (11). These equations verify that at each node the inflow equals the outflow. If the water supply system model has no existing tank in the beginning, there is at least one reference node to set an initial hydraulic head, see constraint (12). With the initial hydraulic head the hydraulic heads of all other nodes are determined. The most important equation to do so is the already introduced head loss equation, which can be found in constraint (13). The bounds of all variables are set in constraints (14)-(17).

Due to the presence of binary variables and the nonconvex quadratic head loss equation in constraint (13), the proposed optimization model becomes a nonconvex Mixed Integer Quadratically Constrained Program (MIQCP).

4 Solution of Optimization Model

The optimization model proposed in Section 3 is a nonconvex MIQCP due to the head loss equation in constraint (13) and the presence of binary variables in (15).

There are different approaches to solve such a nonconvex MIQCP. In this paper a piecewise linearization of the head loss equation is used. Therefore, the model becomes a Mixed Integer Program (MIP) and can be solved via standard MIP solvers, e.g. CPLEX or GUROBI.

There a different formulations to model a piecewise linearization. The formulation used in this paper is described in the following. For further references see for example Padberg (2000) or Dantzig (1966, pp. 547-555).

Let $f: R^n \longrightarrow R$ be any separable nonlinear function, so that the function can be written as $f(x_1, \dots, x_n) = \sum_{j=1}^{n} f_j(x_j)$ with $f_j: R \longrightarrow R$. Let D_j denote the domain of the function f_j. Then D_j can be divided into m different intervals with m starting points $a_k^j,\ k = 0, \dots, m-1$, and one end point a_m^j. Thus, each f_j can be divided into m segments with $b_k^j = f_j(a_k^j),\ k = 0, \dots, m$. This yields $(m+1)$ different coordinates $(a_k^j, b_k^j),\ k = 0, \dots, m$, of function f_j, $j = 1, \dots, n$. If we connect all adjoining coordinates by linear functions, we gain a piecewise linear approximation of f_j. In the following we will omit the index j to get a better overview.

To use this formulation we have to introduce m continuous variables $z_k, k = 1, \dots, m$, and $(m-1)$ binary variables $y_k, k = 1, \dots, m-1$. Then we can state the formulation of a piecewise approximation of function f:

$$
\begin{aligned}
x &= a_0 + \sum_{k=1}^{m} z_k \\
f(x) &= b_0 + \sum_{k=1}^{m} b_k \cdot z_k \\
z_1 &\leq a_1 - a_0, \quad z_m \geq 0, \\
z_k &\geq (a_k - a_{k-1}) y_k, \quad k = 1, \dots, m-1, \\
z_{k+1} &\leq (a_{k+1} - a_k) y_k, \quad k = 1, \dots, m-1, \\
y_k \in \{0, 1\}, &\quad k = 1, \dots, n-1.
\end{aligned}
$$

This formulation has the necessity of introducing additional continuous and binary variables. The more accurate the approximation of function f should be the more intervals of the domain are needed. Thus, there is need of more additional continuous and binary variables which affect the computational time that is needed to solve the model.
In the optimization model introduced in Section 3 the function to be linearized is the head loss equation. As this equation is needed for each pipe and each time step, the number of binary variables is growing with the number of pipes and the number of time steps. With that the computational time for solving the model increases as well. Hence, when considering a real water supply system, the optimization model often cannot be solved in reasonable time. To overcome this fact it seems reasonable to try to reduce the complexity of the optimization model. One idea is to reduce the size of the network before building the optimization model which is pointed out in the next Section.

5 Reduction of water supply system models

The complexity of the linearized optimization model stated above grows with the number of pipes and the number of time steps. So it seems reasonable to reduce the number of pipes in the water supply system model. The reduction should lead to a network model with similar hydraulic properties. There are different techniques to reduce network models (Burgschweiger et al. 2009, pp. 21-24; Maschler and Savic 1999, pp. 19-58). In the developed reduction tool (Pischel 2012) three different techniques are implemented. These techniques are discussed in the following Sections.

5.1 Elimination of end nodes

The first technique to apply is the *elimination of end nodes* and is also known as *elimination of tree structures* (Maschler and Savic 1999, p. 21). An end node is a node which has only one connected pipe. Such an end node can be just removed from the network model. If the end node has a demand of water, the demand is added to the node that lies in front of the end node. Therefore, the head and the flow in the remaining system stay the same. This technique can be applied iteratively until there are no more end nodes. It should be noticed that a reservoir node or a tank node can be end nodes as well. Those nodes are excluded from the elimination process.

This technique leads to a reduced network model that has the same hydraulic properties as the original network model. An illustration of this technique can be found in Figure 2.

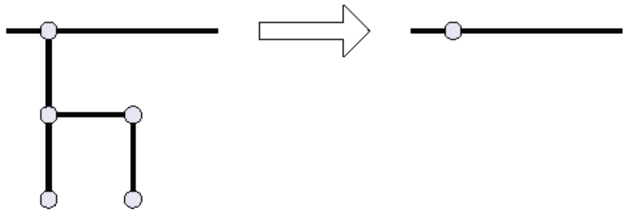

Fig. 2 Elimination of end nodes (Maschler and Savic 1999, p.19)

5.2 Elimination of pipe sequences

The second technique is called *elimination of pipe sequences.* A pipe sequence is a sequence of two pipes connected by one node that is only connected to those two pipes. The pipes in the sequence can be eliminated and replaced by one fictitious pipe. The node in the sequence is eliminated as well. There are two different cases to consider when eliminating the node: the node has no demand of water or it has a demand of water. If the node has no demand, the node and the pipe sequence can be removed and replaced by a new fictitious pipe. If the resistance coefficient of the fictitious pipe is defined as the sum of the resistance coefficients of the removed pipes, the hydraulic properties of the reduced network model remain the same as in the original network model (Burgschweiger et al. 2009, p. 22). If the node has a demand of water, the situation is more complicated. To sustain similar hydraulic properties the demand must be added to the start and end node of the sequence. There are different possibilities to do so. A good way to distribute the demand of the eliminated node is to distribute it depending on the friction losses of the removed pipes. This technique is described in Burgschweiger et al. (2009, pp. 22-23). Here, the hydraulic properties of the reduced network model may differ from those of the original network model. For further details, see Burgschweiger et al. (2009, p. 22) or Maschler and Savic (1999, p.41). If there is a sequence of more than two pipes the technique can be applied iteratively. The main idea of the technique is visualized in Figure 3.

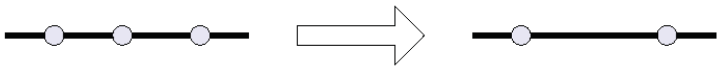

Fig. 3 Replacing pipe sequences by one single pipe (Maschler and Savic 1999, p.19)

5.3 Elimination of parallel pipes

The third implemented technique is called *elimination of parallel pipes*. Parallel pipes are pipes which have the same start and end node. Those pipes can be eliminated and replaced by one fictitious pipe. The head loss between the start and end node of the parallel pipes should remain the same when the parallel pipes are replaced. Thus, the fictitious pipe must have a specific length, diameter and resistance coefficient. If the friction factors of the eliminated pipes are calculated with the law of Prandtl-Kármán (Karger et al. 2008, p. 224), the reduced network model has the same hydraulic properties as the original network model (Burgschweiger et al. 2009, p. 22). Details of how to calculate the new properties of the pipe can be read in Burgschweiger et al. (2009, p. 22), Boulos et al. (2006, pp. 5-5 - 5-8) or Maschler and Savic 1999, pp. 43-45). A visualization of this technique can be seen in Figure 4.

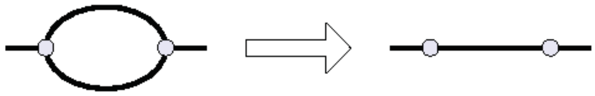

Fig. 4 Replacing parallel pipes by one single pipe (Maschler and Savic 1999, p.19)

5.4 Reduction Tool

The three techniques discussed above are implemented in the reduction tool. It is reasonable to apply those three techniques iteratively, because after the elimination of end nodes there may arise new cases of pipe sequences and parallel pipes. Furthermore, after eliminating parallel pipes there may be new cases of pipe sequences and vice versa.

After applying the three techniques iteratively the reduction tool provides a reduced network model. The hydraulic properties of this model may differ from those of the original network model which is mainly caused by the technique *elimination of pipe sequences*. This *hydraulic error* may lead to infeasibilities if the solution of the optimization model is considered in the original network model. To make sure that the optimal solution still holds when transferring it to the original network model, both the network model and the optimal solution are transferred to a hydraulic simulation tool, which can validate the feasibility of the solution. This process is described in the next Section.

6 Simulation and Modification

In this paper simulation means a numerical simulation that calculates the hydraulic properties in a water supply system such as flows and hydraulic heads. It was decided to use a water simulation tool, which was recently developed by our industry partner Rechenzentrum für Versorgungsnetze Wehr GmbH. This tool is able to calculate flows and heads of a water or gas network during all time periods and takes into account a lot of components that may appear in a water supply system such as pipes, pumps, valves, reservoirs and tanks. Though not considered in the optimization model, it is also possible to define network based rules. Thus, the simulation tool takes into account more details as the optimization model.

The simulation tool receives the original water network model, i.e. before it was reduced by the network reduction tool. The information obtained during the optimization process is transferred to the simulation tool as well. This means that there may be a few new tanks and some tanks may have closed or differ in size. After putting together those information the hydraulic simulation tool runs a simulation. Therefore, the flows and heads are calculated in the original network model with all kinds of components and the new information about the tanks. As the optimal solution was obtained in a reduced network model, it may be infeasible if considered in the original network model, e.g. the size of a tank may be too small. If the simulation tool detects any infeasibility in the original network model with the new tank information, the current solution cannot be optimal for the tank optimization problem. The infeasibilities are caused by the differences of the granularity in the reduced network model in the optimization model and the original network model in the simulation tool. To avoid the infeasibilities their locations in the network are determined. At those locations the reduction techniques are reversed to recover the original structure. The result is a new reduced network model with some parts that have the original structure. The new reduced network model is then transferred to the optimization model and is optimized again. The modification is performed as long as the simulation tool detects any infeasibility. If there are no infeasibilities, the optimal solution leads to correct heads and flows in the original network and the decision support system obtained a satisfying solution for the tank optimization problem.

7 An Approach for a Decision Support System

In Sections 3-6 an approach for a decision support system was presented, which solves the optimization problem of tank usage in water supply systems by a combination of network reduction, mathematical optimization and hydraulic simulation. In this Section this approach is summarized. To capture the main idea of the approach, take a look at Figure 5.

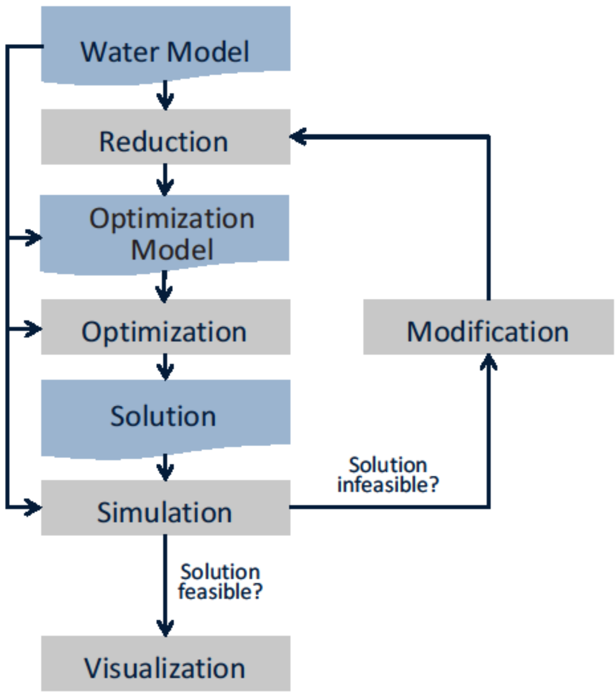

Fig. 5 Overview of approach

As a start, a model of a water supply system is needed. This model contains the topology of the network, the components of the water supply system with their specific properties and some further details, e.g. the information of the inflow and outflow at all time steps and the information where tanks can be built. In addition, the model must store parameters which are needed during the optimization process such as the investment and operational costs of the tanks. Furthermore, there is need of some configuration parameters for the reduction tool, the optimization tool and the simulation tool. All these information are gathered in a so called *Water Model*. Parts of this Water Model are transferred to the reduction tool. These parts include the topology of the network, the components of the water supply system with their specific properties as well as the reduction parameters. The reduction tool then reduces the size of the network model by manipulating the topology of the network. Therefore, end nodes are eliminated, parallel pipes are replaced by one single pipe and pipes in sequences are replaced by one single pipe as well, cf. Section 5. The output is a reduced network model that is combined with the necessary parameters for the optimization and becomes a mathematical optimization model, cf. Section 3. This model is then solved to obtain an optimal solution, if one is available, cf. Section 4. This optimal solution is transferred to the simulation tool, which also receives the original network model that will be combined with the optimal solution. That means that some new tanks are added to the system and some existing tanks are closed or change their sizes. As the original network model takes into account more details, the optimal solution can be considered in a more detailed context. The simulation tool runs a hydraulic simulation, cf. Section 6. If the simulation run was successful, the current solution is feasible and can be visualized. If the simulation tool detects any infeasibility, e.g. the pressure at a node was negative or a tank was too small, then the next step is a modification, cf. Section 6. This modification increases the granularity of the parts of the network model where the infeasibilities arose. Thus, the hydraulic error which was made in those parts of the network model during the reduction process can be prohibited. Then, the optimization model is built and solved again. This process is repeated until the solution is verified by the simulation tool.

8 Summary

In this paper an approach for a decision support system was proposed. The approach combines network reduction, mathematical optimization and hydraulic simulation to solve the optimization problem of optimizing the usage of water tanks in water supply systems. The optimization model is a nonconvex MIQCP and is solved by using a piecewise linearization of the head loss equation for pipes such that the model becomes a MIP. As this linearization requires a lot of binary variables, the model is hard to solve and needs a lot of computational time when considering real water supply systems. To overcome this fact the number of pipes in the model is reduced and therefore the number of binary variables is decreased as well. This leads to reasonable computational times for solving the optimization model. By reducing the number of the pipes there may occur some difference in the hydraulic properties of the reduced model and the original model. To make sure that the obtained optimal solution is still feasible in the original network model a simulation tool validates the solution.

As the optimization model is a nonconvex MIQCP and there are different levels of granularity in the optimization model and the simulation model it cannot be guaranteed that the solution that is obtained by this approach is globally optimal. However, it may give planners of even large water supply systems a good advice where and how to build water tanks.

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