Liner Network Design under Consideration of Transit Times and Partner Networks

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ABSTRACT

This paper presents an evolutionary algorithm for the network design problem in the liner shipping industry with focus on the fitness calculation. The main contribution of this work are several new business constraints, such as committed partner capacities using a maximum flow algorithm and the transit times using a shortest path evaluation. Beside the cargo flows, the transit times are important parameters for the network design and play a major role when selecting a carrier. The algorithm uses a multi-commodity flow network with different layers and linearized bunker costs to calculate the overall network’s profit.

Keywords: Liner Shipping Network Design, Cargo Allocation Problem, Transit Time, Alliances

1 INTRODUCTION

Liner shipping is a major transportation mode that carries about 16% of the worldwide trade volume (UNCTAD 2012). The main advantages are the relatively cheap unit costs due to large container vessels on the one hand, and relatively little environment impact per ton on the other hand.

The liner shipping network design defines the liner services in a strategic or tactical planning horizon. A service is a sequence of ports that performs a round trip. Additionally, vessels are deployed on the service, such that a weekly port call frequency is obtained. The liner shipping network design problem optimizes the port sequences and the deployed capacities. The underlying cargo allocation problem determines the optimal container routing through the network.

Shippers choose the carrier based on different criteria such as costs and transit times (Notteboom and Rodrigue 2008, Gelareh et al 2010). In scope of this paper, transit time is defined as the duration it takes to transport a container between a pair of ports. It is influenced by the port call duration and each liner service’s speed. The storage duration at ports is not considered since it depends on successive planning problems that define the vessel system’s exact schedule based on berthing windows.

The presented algorithm integrates different research areas: The liner shipping network design, the cargo allocation problem and the speed optimization. Therefore, in the remainder of this section, we give a short overview about several publications in these areas. There exists an increasing amount of publications on liner network design such as Argarwal and Ergun 2008, Alvarez 2009, Reinhardt and Pisinger 2011, Mulder and Dekker 2012 or Brouer and Desaulniers 2012. For a more complete overview, the reader is referred to Kjeldsen 2011. To the best of our knowledge, none of the publications considers transit times when optimizing liner shipping networks without predefined transhipment ports.
The cargo allocation problem can be seen on one hand as a method to evaluate liner shipping networks, on the other hand it defines the cargo flows in a tactical horizon and thus defines the transhipment ports. Several studies deal with the cargo allocation problem. Some focus on the repositioning of empty containers, since the global repositioning due to trade imbalances is estimated to 15 billion US$ in 2002 which is about 27% of the total world fleet operational costs (Song et al 2005). Brouer et al 2011 develop a column generation approach to solve the cargo allocation problems in large scale instances. They include empty container repositioning, container leasing and container unit costs. Song and Dong 2012 consider inventory holding, transhipment, transportation, backlog and demurrage costs, but do not include bunker costs or alliances. For an overview on empty container repositioning publications the reader is referred to Brouer et al 2011. Recently, Wang et al 2013 published a container routing model that incorporates transit time requirements between the cargo flows’ origin and destination. Wang and Meng 2013 analysed reversing the port rotation and included transhipment, slot purchasing and inventory costs. To the best of our knowledge, the cargo allocation problem in the tactical planning horizon with complex route types, alliances and exact speed calculation has not been addressed before.

The third research direction that this work incorporates is the speed optimization. One of the first works on speed optimization was published by Brown et al 1987. They scheduled crude oil vessels but did not consider transhipment. Fagerholt et al 2009 and Norstadt et al 2011 optimized vessels’ speed under consideration of time windows in tramp shipping which might be applicable for the liner shipping industry by including time windows at the journey’s last port. However, container transhipment and port call durations are not included in these models. Recently, Wand and Meng 2012 published a model to optimize the speed per leg in liner services that included port call durations but the container routes where given externally.

2 PROBLEM DEFINITION

A liner shipping carrier’s network consists of a set of liner services that can be either operated by the carrier itself or by a partner. A service $i$ is defined as a list of edges between ports that perform a round trip $S_i = (o_1, d_1), (d_1, d_2), ..., (d_{n-1}, o_1)$ and have a maximum duration for each roundtrip. Each Service has a specific number of vessels $VC_s$ of a specific type $VT_s \in VT$ deployed. Since most of the services are operated on a weekly basis (see Stopford 2009), we focus on a weekly frequency leading to a maximum round trip duration of $VC_s \times 7$. Other frequencies could be incorporated easily, but would lead to a further set of decisions that have to be made.

In this section, the extensions of the existing cargo allocation problem are presented, namely the portcall duration and resulting bunker cost calculation, deadweight scales, alliances and complex route types.

2.1 Port call duration and bunker costs calculation

In figure 1, two liner services $S_1 = ((P_1, P_2), (P_2, P_1))$ and $S_2 = ((P_2, P_3), (P_3, P_2))$ are shown. Service 1 is operated by the liner carrier, whereas Service 2 is operated by a partner carrier. The legs’ distances in service 1 are assumed to be 3400 nautical miles per leg. Thus, without any duration at the ports, the two vessels must cruise with a speed of at least $s = 3400 \text{NM} / (24 \times 7D) = 20.24$ knots to fulfil the weekly service frequency. The two vessels deployed on service 1 cruise the whole 14 days of the round trip at this speed, since no port duration is considered so far.

Typically, a container vessel needs several hours to enter or leave the terminal, which is referred as tug boat pilot time, for example 2.5 hours per port call (no delay due to port
congestion or missed berth time windows is considered). Furthermore, loading and unloading each container also takes time. Practical data indicates a container terminal throughput of 10 – 40 containers per hour. Thus, moving 500 containers at 30 moves per hour results in the port call duration of $16 + 5 = 21$ hours including pilot time. This reduces the available time to keep the weekly service frequency to 6,125 days and leads to an increased speed of 23.13 knots. The increased speed usually results in higher bunker cost that is approximated in the literature by a cubic cost function (see Stopford 2009). We fitted the practical bunker profiles with a polynomial of grade three using linear least squares. The polynomial approximates the lower speeds better than the cubic function which is important due to increased carriers’ slow steaming strategies.

Figure 1. Two liner services, one operated by the carrier, the other by the partner

2.2 Deadweight scales

Another extension to previous works is the use of deadweight scales. Many ports have a maximum draft at high tide where ships can enter. In the scope of this paper, only drafts at high tides are considered. Previous publications have matched the port vessel type compatibility by using a fixed draft (see for example Alvarez 2009). Due to increased vessel size and port draft restrictions, network planning departments must consider the compatibility in a more detailed way. The draft constraints can change the port rotation to ensure high utilizations on long haul legs. A typical nominal 4300 TEU capacity vessel type has a light ship draft of 9 meters, matching the compatibility. The vessel’s draft increases due to the loaded deadweight (compare table 1) and restricts the loaded volume in a port.

Table 1. Exemplary deadweight scale for a 4300 TEU vessel

<table>
<thead>
<tr>
<th>Deadweight (t)</th>
<th>Draft (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>42188</td>
<td>9</td>
</tr>
<tr>
<td>59896</td>
<td>11</td>
</tr>
<tr>
<td>77065</td>
<td>13</td>
</tr>
</tbody>
</table>

The vessel’s deadweight scales can be linearized in a fairly good way. The provided practical deadweight scales lead to a coefficient of determination ($R^2$) of 0.99 – 1, indicating a reasonable linear regression’s approximation.

2.3 Slot swaps and resource groups

The third extension to existing publications is the consideration of slot swaps. Most of the liner carriers cooperate with other carriers to extend their network. In such an alliance (or consortia), carriers agree on a specific cargo volumes on specific service legs. In scope of this paper, only slot swaps are considered. Slot swaps allow the cargo transportation on the partner services without any slot costs. However, the agreed capacity must be served for the partner in the own optimized networks too. The solution approach ensures the partner volumes can be transported between the agreed ports.

Container carriers transport different standardized containers. In this work, only TEU and FFE, both dry and reefer, are considered but the modelling approach allows additional container types easily. We define a services’ capacity by using resource groups. Each resource group consists of different resources that can be utilized by the transported cargo flows and
empty containers. For instance, table 2 presents capacities for a service operated by the carrier itself.

Table 2. Exemplary service’s capacities due to a vessel deployment

<table>
<thead>
<tr>
<th>Resource Group</th>
<th>Capacity</th>
<th>Shares capacity among resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nominal Capacity</strong></td>
<td>1000 dry TEU @14 tons 100 Reefers</td>
<td>/</td>
</tr>
<tr>
<td>TOTAL_SLOTS</td>
<td>1100 TEU Slots</td>
<td>Dry_Slot + Reefer_Slot</td>
</tr>
<tr>
<td>REEFER_SLOTS</td>
<td>100 Slots</td>
<td>Reefer_Slot</td>
</tr>
<tr>
<td>MAX_TON_DWT</td>
<td>14,000 Tons</td>
<td>Weight</td>
</tr>
</tbody>
</table>

However, when using partners, capacity constraints can be defined per disjunct set of legs, called segments SG, and resource groups RG_sg to distinguish different trades (such as north or south bound).

Note that the concept of resource groups can also be used to model constraints on width, height or length that are relevant in project shipping.

2.4 Complex route types

A liner shipping network consists of a mix of different route types (Notteboom 2006). Although mentioned by different authors, such as Reinhardt and Pisinger 2010 and Jepsen et al 2011, complex route types have gained relatively low attention from the research community when optimizing networks. In figure 2 the four possible route types in liner shipping are presented: Pendulum routes, that consists of two ports, circle routes where no port is visited twice in a round trip, butterfly routes where exactly one port is visited twice and conveyor belt routes where two or more ports are visited twice. Notteboom and Rodrigue 2008 state that modern networks consist of highly interwoven services with different route types. We also found this combination in practice, where the circle and conveyor belt routes are used most often. One advantage of the complex routes is that ports can be called again to transport interregional demand, freeing capacity before serving a long haul leg between regions and help fulfilling transit time requirements on long distances.

![Figure 2. Different route types: Pendulum, circle, butterfly and conveyor belt routes](image)

To sum up, the solution method considers the vessel speed optimization, deadweight scales, slot swaps, complex route types and empty container repositioning. To the best of our knowledge this combination has not been considered in the container allocation problem before.

3 EXTENSION OF THE CARGO ALLOCATION PROBLEM

In this section the cargo allocation model, which is part of the overall solution approach of the network design problem, is presented. The model extends current formulations by the introduced aspects from section 2. The integration into an evolutionary algorithm is explained further in section 4.
3.1 Layered network formulation

The main modelling challenge when using complex route types is to distinguish the different calls of the same port to ensure correct transhipment and cargo flow volumes. Reinhardt and Pisinger 2010 use port call numbering by the model, if the port is called a second time. This formulation works well for butterfly routes, however it seems to be difficult to use this approach for conveyor belt routes. Thus, this work uses a layered approach to formalize the complex route types, which has been presented in Agra et al 2012 (without transhipment) and Guericke et al 2012 (with transhipment). So far, a liner service consists of an ordered tuple of edges between ports \( S_i = ((o_1, d_1), (d_1, d_2), ..., (d_{n-1}, o_1)) \) (see figure 4). The service calls the ports \( P1, P2, P3, P2 \) and \( P1 \) and is thus called a butterfly service. The problem of calculating the exact transhipment volumes appears at port \( P2 \). To distinguish the port call, it is transformed into a layered formulation by adding two new indices \( l \) and \( l' \) to all legs \( (o_i, d_i, l_i, l_i') \). The indices represent the origin and destination layer. To transform the liner service, one starts at layer \( l=l'=1 \) (one) and increases the destination layer \( l' \), if a port is called twice on the current layer \( l \). The resulting network is visualized in figure 4. Each port at each layer can be used as a transhipment port to other services. Furthermore, cargo can be unloaded at layer 1 and loaded at layer 2. The resulting liner service definition is \( SL_i = ((o_1, d_1, l_1, l_1'), (d_1, d_2, l_2, l_2'), ..., (d_{n-1}, o_1, l'_{n-1}, l_1)) \). The other multi-commodity network layers are the liner services and each commodity (i.e. each equipment type for the empty container repositioning and each cargo flow). With this formulation more than two port calls of the same port per round trip could be modelled. However, this is very rare in practice and thus not allowed in this work.

![Figure 4. Non-layered and layered liner service](image)

3.2 Non-linear cargo allocation model

In this paragraph, an exact formulation for the cargo allocation problem with empty container repositioning, complex routes, alliances and speed optimization is presented. The model assumes a steady state and is formulated as a multi-commodity flow network. Thus, no transformation between services is considered. Let \( S \) be set of liner services in the carrier’s network design, whereas \( S \) is composed of the pairwise disjunct sets of own services \( S^O \) and the slot sharing \( S^{PSS} \) services from the partners. Let \( P_s \subseteq P \) be the set of ports, called by liner service \( S \). Each port \( p \) has a throughput in moves per hour \( tp_p \) and a maximum draft at high tide \( D_p^{Max} \). \( L_S \) is an ordered set of the arcs, used by liner service \( S \), \( \Gamma_S \) the set of layers. Each service has a capacity associated, \( C_{rg} \) for own, \( C_{rg,pg} \) for partner services. Each vessel type’s deadweight scale is given by the slope \( dw_{vts}^{Slope} \) and interception \( dw_{vts}^{Intercept} \). \( N \) is the set of cargo flows with elements of type \( n = (o_n, d_n, e_n, r_n, q_n) \in N \), whereas \( o_n \) is the cargo flows origin, \( d_n \) the destination port, \( e_n \in ET \) the equipment type (of a specific container type and size), \( r_n \) the revenue per unit and \( q_n \) the estimated servable quantity in the planning horizon. Furthermore, empty container types \( ET \) and cargo flows \( N \) are denoted as commodities \( C = ET \cup N \). Each commodity utilizes different resources \( u_{rc} \), as described in the previous sections. \( VT \) are the available vessel types, \( RG \) denotes the resource groups, \( R \) the set of
resources, \( RU_{c,rg} \) the utilized resources of resource group \( rg \) by commodity \( c \). \( SG_S \) a set of disjunct leg tuples, with a capacity per resource group of \( RG_{sg} \). For each liner service the number of service round trips in the planning horizon \( RT_s \) can be calculated in advance. Given a service frequency (of typically 7 days) and a number of vessels, the round trips are \( RT_s = \frac{\tau}{(VC_s \times 7)} \), where \( \tau \) is the planning horizon length in days.

The following decision variables are used: \( a_n \) is the served volume of cargo flow \( n \). \( l_{s,c,p,l} \) and \( u_{s,c,p,l} \) is the loaded and unloaded volume of commodity (laden or empty container) \( c \) in liner service \( s \) at port \( p \) at layer \( l \). \( x_{s,c,i,j,l,l'} \) is the transported amount of commodity \( c \) from port \( i \) to \( j \), from layer \( l \) to layer \( l' \) by using service \( s \). \( k_s \) is the speed in knots of service \( s \), valid for all legs on the service. First, a formulation of the multi-commodity flow network with non-linear bunker costs is presented.

\[
\text{max profit} = \sum_{n \in N} r_n a_n - \sum_{s \in S} \left( \sum_{p \in P_s} \phi_{p,VT_s} C_sRT_s + \phi_{VT_s} VC_s \right)
- \sum_{s \in S} \left( \sum_{c \in C} \sum_{p \in P_s} \phi_{cp} (l_{s,c,p,l} + u_{s,c,p,l}) + \sum_{e \in ET} \sum_{n \in N} (e,l) = (e,l') \in E_s \right) \phi_{et} a_n \right)
- e \sum_{s \in S} \left( VC_s \right)
- \sum_{p \in P_s} \left( \sum_{c \in C} \sum_{l \in L_p} \frac{1}{tp} \left( \frac{t_{pin} + t_{pout} + t_{FP} + t_{p}^b}{24} + \frac{\tau}{24} \right) \right) bc(k_s)
\]

The objective (1) of the model is to maximize the profit in the tactical planning horizon, where profitable cargo can still be selected. The objective consists of the revenue for the served container demand minus the port call costs \( \phi_{p,VT_s} \) for all vessels for each round trip in the planning horizon. Furthermore, depreciation costs for the planning horizon \( \phi_{VT_s} \) for each service’s vessel type, container handling (transhipment) costs \( \phi_{cp} \), container depreciation \( \phi_{et} \) and bunker costs are used. The bunker costs are calculated by the consumption per day of the optimal speed \( bc(k_s) \) times the costs per ton \( e \) times the sea duration in the planning horizon. This duration depends on the overall port call time. Transhipment, bunker, port call and vessel depreciation costs within partner services are paid by the partner. Inventory costs at ports are not considered in this work since the terminals often provide several days of free storage for transshipment containers. The objective is subject to the following constraints.

\( \forall s \in S, c \in C, p \in P_s, l \in \Gamma_s \):

\[
\sum_{(i,p,l',l) \in E_s : l - 1 \leq l' \leq l \forall (l = \text{last}(\Gamma_s) \land l = 1)} x_{s,c,i,p,l',l} + l_{s,c,p,l} = \sum_{(p,j,l') \in E_s : l \leq l' \leq 1 \forall (l = \text{last}(\Gamma_s) \land l' = 1)} x_{s,c,p,j,l,l'} + u_{s,c,p,l}
\]

Constraint 2 ensures the flow conservation between different services and ports for all commodities.

\( \forall p \in P, n \in N \):

\[
\sum_{s \in S} \sum_{l \in L_p} l_{s,n,p,l} = \sum_{s \in S} \sum_{l \in L_p} u_{s,n,p,l} + \begin{cases} +a_n, & \text{if } p = o_n \\ -a_n, & \text{if } p = d_n \\ 0, & \text{else} \end{cases}
\]
At each port, cargo flow can be picked up, unless their destination $d_n$ is reached. At a commodity’s origin $o_n$, the containers enter the network.

\[ \forall p \in P, et \in ET: \sum_{s \in S} \sum_{i \in I_s} l_{s,et,p,i} = \sum_{s \in S} \sum_{i \in I_s} u_{s,et,p,i} + \left( \sum_{n \in N: et = et \land d_n = p} \alpha_n - \sum_{n \in N: et = et \land o_n = p} \alpha_n \right) \tag{4} \]

Constraint 4 ensures the repositioning of empty containers that is calculated by the served cargo flow balance at each port.

\[ \forall s \in S, sg \in SG_s, (i, j, l, l') \in sg, rg \in RG_{sg}: \sum_{c \in C} \sum_{r \in RU_{crg}} x_{s, c, i, j, l, l'} u_{r, c} \leq C_{r, g, sg} VC_s RT_s \tag{5} \]

The capacity for each liner service’s leg equals the capacity for each resource group (with can be either the vessel’s capacity or the shared slots) times the number of vessels times the round trips in the planning horizon.

\[ \forall s \in S^0, p \in P_s, l \in I_s: \sum_{c \in C} \sum_{(i, p, l', l) \in L_s} x_{s, c, i, p, l'} u_{weight, c} d w_{VT_s}^{slope} + d w_{intercept}^{VT_s} \leq D_{p}^{max} VC_s RT_s \tag{6} \]

\[ \forall s \in S^0, p \in P_s, l \in I_s: \sum_{c \in C} \sum_{(p, j, l, l') \in L_s} x_{s, c, p, j, l, l'} u_{weight, c} d w_{VT_s}^{slope} + d w_{intercept}^{VT_s} \leq D_{p}^{max} VC_s RT_s \tag{7} \]

The linearized deadweight scales bound the number of containers that can enter a port with the carrier’s services. Partner services must ensure the deadweight according to the agreed slots individually. Note that the liner service vessel type’s light ship draft must be compatible with the port’s draft at high tide to avoid additional variables.

\[ \forall s \in S^0: k_s = \frac{\sum_{l \in I_s} dist_i}{7 \cdot VC_s - \sum_{p \in P_s} \left( \sum_{c \in C} \sum_{l \in I_s} (u_{s, c, p, l} + l_{s, c, p, l}) \frac{1}{t_p} \frac{1}{24} + \left( t_{P}^{in} + t_{P}^{out} + t_{P}^{in} + t_{P}^{out} \right) \right)} \tag{8} \]

The last constraint calculates the speed of each service. It is calculated by taking into account the loaded and unloaded containers, the pilot in and out hours, the duration caused by partner containers plus a tactical buffer time.

The variables are subject to the following bounds:
- $\forall s \in S, c \in C, p \in P_s, l \in I_s: u_{s, c, p, l}, l_{s, c, p, l} \geq 0$
- $\forall s \in S, c \in C, (i, p, l', l) \in L_s: x_{s, c, p, j, l, l'} \geq 0$
- $\forall n \in N: 0 \leq \alpha_n \leq q_n$
- $\forall s \in S^0: k_s \geq 0$
3.3 Linearization of bunker costs

The non-linear bunker costs can be linearized by using binary variables, as described in Padberg 2000. For each own liner service, a discretization level of $\delta$ is chosen, leading to $\delta \times |S^o|$ many zero-one variables. For each liner service $s$, the variables $z_{s,0}, z_{s,1}, \ldots, z_{s,\delta}$ and binaries $y_{s,1}, y_{s,2}, \ldots, y_{s,\delta}$ are introduced into the model, used in the following three intervals activation constraints.

$$\forall s \in S^o: z_{s,0} \leq a_{s,1} - a_{s,0}$$
$$\forall 1 \leq i \leq \delta - 1, s \in S^o: z_{s,i} \geq (a_{s,i} - a_{s,i-1}) y_{s,i}$$
$$\forall 1 \leq i \leq \delta - 1, s \in S^o: z_{s,i+1} \leq (a_{s,i+1} - a_{s,i}) y_{s,i}$$

(9)

The bunker cost function is modelled as a function of the discrete port call duration $a_{s,i}$ per service. With help of the constraints, the $y_{s,i}$ are successively activated to reach the required port call duration (PCD). The non-negativity constraints are as follows:

$$\forall 0 \leq i \leq \delta, s \in S^o: z_{s,i} \geq 0$$
$$\forall 1 \leq i \leq \delta, s \in S^o: y_{s,i} \in \{0,1\}$$

To calculate the PCD, following formula can be used:

$$\forall s \in S^o: PCD_s = a_{s,0} + \sum_{i=1}^{\delta} z_{s,i}$$

(10)

By having the PCD, the bunker cost function is calculated by equation 11, where $b_{s,i}$ are the bunker costs for the port call duration of $z_{s,i}$:

$$\forall s \in S^o: \phi_s^{Bunker} = b_{s,0} + \sum_{i=1}^{\delta} \left( \frac{b_{s,i} - b_{s,i-1}}{a_{s,i} - a_{s,i-1}} \right) z_{s,i}$$

(11)

The PCD depends on the loaded and unloaded volume of laden and empty containers, transported by the carrier himself, the tug boat pilot in $t_{p,In}^p$ and out time $t_{p,Out}^p$, the partner duration $t_{p,\text{p}}$ and the buffer time $t_{p,B}$. Note, that the unloading and loading cargo volume is defined for the whole planning horizon $\tau$, where the pilot times and partner duration are per port call. Thus, the number of roundtrips and number of vessels have to be multiplied. This leads to the required PCD for each service $s \in S^o$ of

$$PCD_s = \sum_{p \in P_s} \sum_{t \in T_s} \sum_{c \in C} (u_{s,c,p,t} + l_{s,c,p,t}) \frac{1}{tp} \frac{1}{24}$$
$$\quad \quad \quad + \left( \sum_{(i,l,l',t') \in L_s} (t_{p,In}^p + t_{p,Out}^p + t_{p,FP}^p + t_{p,B}^p) \right) VC_s RT_s \frac{1}{24}$$

(12)

The objective’s bunker cost term can be replaced by $\phi_s^{Bunker}$, the speed calculation in equation (8) by constraints (9), (10) and (12). A further simplification can be made by defining the $y$ variables to be of $[0;1]$.

4 EVOLUTIONARY ALGORITHM

In the following section the evolutionary algorithm, which is using the cargo allocation model for calculating the revenue, is presented.
3.1 Calculating a network’s fitness

The network’s fitness is used in the evolutionary algorithm to compare the quality of solutions. For each network the profit is calculated by solving the cargo allocation problem (see section 3). Beside the network’s generated profit, successive costs are imposed to avoid further complex business constraints directly in the model. First, the transit times in the evaluated network are verified by calculating the shortest path between a set of port tuples \((i,j)\) using Dijkstra’s algorithm (Dijkstra 1959), whereas the edge costs are the port durations (pilot in and out, partner, loading/unloading and buffers) and the sea duration on the service’s edge. Note that we currently do not consider increasing the speed to fulfil the transit times. The second constraint is the consideration of committed partner volumes on specific legs \((i,j)\). Using the output of the model above, a maximum flow algorithm (Ford and Fulkerson 1956) is used between port \(i\) to \(j\) using the capacities of the provided TEU capacity (by the vessel type or the agreed partner capacity) minus the TEU flow. Thus, a maximum flow of the residual network’s capacity is searched and penalty costs imposed when the maximum flow is smaller than the agreed amount. Afterwards, the fitness used for the evolutionary algorithm is calculated using the profit minus penalty costs for invalid transit times, not fulfilled partner capacities and disallowed port combinations due to political regulations (such as Liberia and Israel). As proposed by Richardson et al 1989, we use a gradual penalty function of the amount of invalid business constraints.

3.2 Overview of the algorithm

The presented evolutionary algorithm uses a standard workflow, shown in figure 3. The initial population is generated using random networks. These networks are created by a mix of pendulum route networks and greedy networks. The greedy heuristic creates services by following the cargo flow structure until a round trip is performed. The networks are evaluated using the fitness calculation described in section 3.1. Afterwards, networks (called parents) for the crossover operation are chosen by a binary tournament selection. Two networks are combined to create a new network, called child. For the crossover operator, we currently use the uniform (UX) operator, such that the new child contains services randomly selected from both parents. The mutation is performed by inserting and deleting random ports from the service and inserting or deleting a (pendulum) service. After the mutations have been applied, a local search is performed that creates 2-optimal liner services regarding the distance. We use a relatively small population size of \(<= 10\) due the long evaluation using the exact model and a small mutation rate of 5%.

Finally, a new population is selected from the parent and child population using elitism. The best 20% networks regarding the fitness function are selected for the next generation. The other 80% are chosen randomly from the remaining candidates whereas duplicates are forbidden.

![Figure 3. Process of the evolutionary algorithm](image-url)
4 NUMERICAL RESULTS

The numerical results are created on a workstation PC with an Intel Core i5 2.6 GHz, 8 GB RAM and Windows 7 x64. The evolutionary algorithm and the cargo allocation problem are implemented in C# 4.0 and visualized in a prototypical web interface that allows changing the services on the fly (see figure 5). For solving the model, Gurobi 5.0.1 with its .NET interface and the dual simplex is used. To speed up the optimization process, the algorithm uses a multi-threaded population evaluation.

Figure 5. Optimization software

The model and the approximation have been tested in 5 test instances, ranging from 10 ports with 100 cargo flows (AFC_P10_D100) up to 100 ports with 10,000 cargo flows (AFC_P100_D10000). The last two instances are practical instance from a global liner carrier without (*_NP) and with (*_WP?) partners. The last two rows in table 3 differ by the initial population. *_WP1 contains the current carrier’s network in the initial population, *_WP2 only randomly generated networks. The instances are a sub network in the Mediterranean region.

The bunker cost discretization relies on a given amount of supporting points. With increasing points, the cost function is approximated more exactly. For further experiments we use 25 supporting points, since it gives an acceptable bunker cost approximation.

<table>
<thead>
<tr>
<th>Instance name</th>
<th>Initial Fitness</th>
<th>Best Fitness</th>
<th>Valid solution after</th>
<th>Reference Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFC_P10_D100</td>
<td>86.14 Mio. $</td>
<td>186.14 Mio. $</td>
<td>No business constraints</td>
<td>-</td>
</tr>
<tr>
<td>AFC_P30_D1000</td>
<td>484.62 Mio. $</td>
<td>1.361 Mio. $</td>
<td>No business constraints</td>
<td>-</td>
</tr>
<tr>
<td>AFC_P50_D1000</td>
<td>-12.30 Mio. $</td>
<td>526.86 Mio. $</td>
<td>No business constraints</td>
<td>-</td>
</tr>
<tr>
<td>MED_P35_D2000_NP</td>
<td>-413.51 Mio. $</td>
<td>26.59 Mio $</td>
<td>0.7 Min.</td>
<td>-</td>
</tr>
<tr>
<td>MED_P35_D2000_WP1</td>
<td>42.36 Mio. $</td>
<td>45.76 Mio. $</td>
<td>0.0 Min.</td>
<td>42.36 Mio. $</td>
</tr>
<tr>
<td>MED_P35_D2000_WP2</td>
<td>-1.39 Mrd. $</td>
<td>36.53 Mio. $</td>
<td>20.5 Min.</td>
<td>42.36 Mio. $</td>
</tr>
</tbody>
</table>

The results of the evolutionary algorithm are presented in table 3. Small random instances can be solved in a reasonable amount of time with large improvements. The results indicate, that the practical instance is much harder to solve. Without using the existing partners (instance MED_P35_D2000_NP), the solution quality is about half the current revenue. When using
partners, the runtime highly depends on the initial solution. In row 5 the initial population is based on the existing network and thus improved networks can be found relatively quickly. However, when starting which random networks, the process takes quite long until valid and good solutions are found. The results indicate that for the given cargo flow and transit time structure the network is already optimized and only small improvements could be found in the given time.

5 CONCLUSION

In this paper an evolutionary algorithm and a practical cargo allocation model has been presented. To the best of our knowledge, transit times have not been included in the liner shipping network design problem without predetermined hub and spokes before. The cargo allocation model uses several practical constraints that can be used to directly get weekly vessel schedules out of the network design, leading to a more integrative solution and acceptance from industry.

As the preliminary numerical results indicate, the meta-heuristic can be used to optimize middle sized practical networks in a reasonable amount of time. However, with increasing network size and low quality initial populations, the random search takes too long to get high quality solutions. Thus, further work has to be done to evaluate the networks using heuristic approaches and start with better initial networks. A first approximation of the cargo allocation problem using a resource constrained shortest path problem found the same solutions for the practical instances in less than 2 minutes. Furthermore, we claim that the problem is a special vehicle routing problem with transhipments, variable speed and capacity constraints so solution approaches from the vehicle routing problem might be applicable. The shortest path and max flow problems should be integrated into the cargo allocation problem to achieve better results.

At the time of the conference we will present further numerical results regarding the network transit times and the effects of complex route types on the network.

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