

# A New Model Approach on Cost-Optimal Charging Infrastructure for Electric-Drive Vehicle Fleets

Kostja Siefen, Leena Suhl, and Achim Koberstein

**Abstract** Electric-Drive Vehicle Fleets need proper charging infrastructure available in the area of operation. The number of potential locations can be quite large with differing costs for preparation, installations, operation and maintenance. We consider a planning approach based on vehicle movement scenarios. Given a large set of vehicle movements and parking activities, we search for a cost minimal subset of all charging locations subject to proper supply of all vehicles.

## 1 Introduction

We consider the perspective of a future operator of an electric drive vehicle fleet. Vehicle engines in our case are solely powered by a permanently installed battery (see [2] for a thorough analysis of recent technologies). The vehicles will be used to serve different mobility demand by the vehicle users, e.g. transport of persons or goods. Movements happen one-way, vehicle drivers may change every time and the set of allowed parking positions can be quite large in urban areas. Fleet characteristics such as the distribution of driving distances, parking positions (and durations) can be completely different between problem instances (see [7]).

To supply energy to all fleet vehicles, a proper charging infrastructure has to be available (see [5, 6]). This is a set of locations with parking space and a technical installation suitable to transfer into the energy carrier of the vehicle. Charging is done with a certain electric power - agreed upon between vehicle and charging spot. The total available power can be different among locations. We do not distinguish between conductive (plug) and inductive (contactless) charging, vehicles just need to stay parked. Nevertheless charging can be started at any given battery level and

---

Decision Support & Operations Research Lab,  
University of Paderborn, Warburger Str. 100, 33098 Paderborn, Germany  
e-mail: [siefen,suhl,koberstein@dsor.de](mailto:siefen,suhl,koberstein@dsor.de)

may as well be interrupted at any time. In our case, we do not consider the flow of electric energy back into the power grid (see [10] for this concept).

A set of potential locations is prepared in advance, and its size is usually quite large, while only a small subset may be needed. Some locations may already be available while others are mandatory to be built. The planner chooses a subset of all available locations, that means a simultaneous decision how many locations are to be opened, where the stations will be located and which capacity is installed.

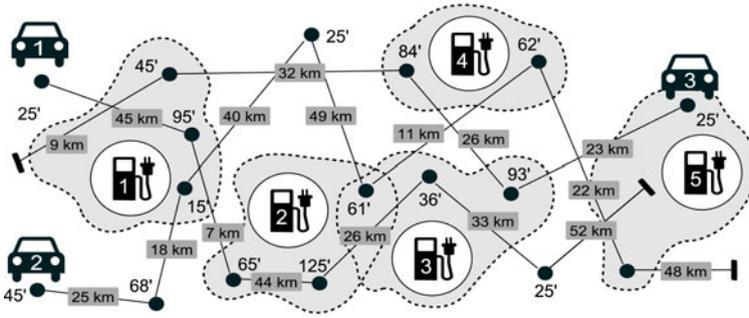
## 2 Problem Description

The choice of charging locations needs to provide enough capacity for all vehicle demands while total costs for installation and operation are to be minimized. The maximum accepted distance between location and vehicle parking position is a planning parameter. If locations are close enough to the vehicle, it can charge its battery with a location-specific power depending on the available parking time. The needed on-site capacity (i.e. total number of available chargers) for each location depends on the amount of simultaneous parking vehicles. Vehicle parking durations may in general be shorter or longer than the time needed to fully charge the battery. Vehicles may have different movement and parking characteristics (i.e. trip distances and parking durations).

Location planning problems in literature (see [3] for a recent survey) usually consider spatially fixed demand points (customers) with a certain need of goods over time. The demand is fulfilled by assigning sufficient quantities to connections between supply points (production sites, warehouses) and consumers. The total costs are derived from the locations investments, the building of capacity and the prices for transportation. These models do not consider the movement of demand points which consume energy.

Coverage problems like ambulance location models (see [1]) choose a cost-minimal set of locations which assure proper response times for the population. Problems like  $p$ -center,  $p$ -median and MCLP maximize the total coverage for a fixed amount of  $p$  locations (see [9]). In our problem domain, we do not have to cover all demand points (vehicle positions). In fact, with small distances and sufficient energy, vehicles may park at many places without charging. Furthermore, the location decision has an impact on the demand. The availability of charging locations in one area can avoid their necessity in other places.

To account for the given aspects, we developed a new model. As planning base we consider a movement scenario of all electric vehicles. Starting from a reference point zero in time, we have the movements of all vehicles with parking activities between. We call this alternate chain of driving and parking of one vehicle a *movement sequence* which consists of many consecutive *sequence elements*. Each sequence element contains the information of parking time and potential supply locations nearby. The possible amount of energy consumption is derived from the parking



**Fig. 1** Illustration of a simple problem instance with 3 vehicles and 5 potential locations. The black points are parking positions with a given parking time. Parking positions can be in range of one or more supply locations (gray areas). Between parking positions, vehicles move a given distance.

time and technological parameters of location and vehicle. We therefore have no knowledge of the exact vehicle routes nor do we decide about it (see [11]).

The set of potential locations is distributed over the region of operation. Locations can have very different cost values for installation, maintenance and operation. A location can be used by a vehicle if it's close enough (e.g. does not exceed a maximal distance parameter). Vehicles and locations must be technologically compatible. A valid solution of the problem is an assignment of sequence elements to locations, such that each vehicle is supplied (remaining range never exceeds the minimal and maximal battery level). Additionally the location capacities have to be sufficient. The location capacities do not simply depend on the total number of assigned vehicles. They are derived from the maximum amount of simultaneously occupying vehicles.

### 3 Mixed-Integer Model Formulation

We start with the set of potential locations  $L$ . For all locations  $l \in L$  we define a set  $K^l$  of discrete capacity levels. Each capacity level  $k \in K^l$  has an amount of available charging slots  $K_{l,k}$ . The total cost value  $C_{l,k}$  is derived from all fixed and variable costs over the projected time of operation (e.g. using the Net Present Value Method, see [8]).

Let  $\sigma_{l,k}$  denote the decision to operate location  $l \in L$  in capacity level  $k \in K^l$ . The planning goal is to find a cost minimal subset of locations and capacities, thus giving the following objective function:

$$\min \sum_{l \in L} \sum_{k \in K^l} C_{l,k} \cdot \sigma_{l,k} \tag{1}$$

Any location may be operated in at most one defined capacity level.  $\sigma_{l,k} = 0$  for all  $k \in K^l$  implies the unavailability of the location in the solution.

$$\sum_{k \in K^l} \sigma_{l,k} \leq 1 \quad \forall l \in L \quad (2)$$

Let  $S$  be the set of all sequences. For each  $s \in S$  we define a set  $I^s = \{1, 2, \dots, I_s\}$  of sequence numbers, i.e. a sequence element is identified by a tuple  $(s, i)$  with  $s \in S$  and  $i \in I^s$ . Each sequence element consists of a set of supply options  $L^{s,i}$ . The locations  $l \in L^{s,i}$  can be used by the vehicle, because they are close enough to the parking position and technologically compatible.

For simplification reasons we measure the vehicle energy level in remaining range (e.g. kilometers or miles). For any sequence there is a known initial remaining range  $X_s^0$ . To model the change of the energy level over time and space we use balance equations between sequence elements.

Let  $q_{s,i,l}$  denote the amount of mileage collected. Remaining mileage at position  $i+1$  is the previous remaining mileage  $x_{s,i}$  minus the driven distance  $D_{s,i}$  plus additional mileage  $q_{s,i+1,l}$  at location  $l \in L^{s,i}$ :

$$x_{s,1} = X_s^0 + \sum_{l \in L^{s,1}} q_{s,1,l} \quad \forall s \in S \quad (3)$$

$$x_{s,i+1} = x_{s,i} + \sum_{l \in L^{s,i}} q_{s,i+1,l} - D_{s,i} \quad \forall s \in S, i \in I^s \setminus \{I_s\} \quad (4)$$

As the vehicle energy carrier is a battery, its level can never fall below zero and may also never exceed the available capacity. This is enforced by bounding the remaining mileage to the battery conditions. An even better bound is the next distance  $D_{s,i}$ :

$$0 \leq D_{s,i} \leq x_{s,i} \leq M_s \quad \forall s \in S, i \in I^s \quad (5)$$

Let  $\tau_{s,i,l}$  denote the assignment of a sequence element to a supply location (indicator variable). Each sequence element may be assigned at most once. For every sequence element  $(s, i)$  we calculate for every allowed location  $l \in L^{s,i}$  the maximal possible gain of mileage  $R_{s,i,l}$  based on the possible charging power, the available time and the vehicle's average power consumption. In fact,  $q_{s,i,l}$  can also be less than  $R_{s,i,l}$  if the remaining range  $x_{s,i}$  equals the battery capacity  $M_s$  (i.e. vehicle is fully charged although there would be time left for additional range).

$$q_{s,i,l} \leq R_{s,i,l} \cdot \tau_{s,i,l} \quad \forall s \in S, i \in I^s, l \in L^{s,i} \quad (6)$$

$$\sum_{l \in L^{s,i}} \tau_{s,i,l} \leq 1 \quad \forall s \in S, i \in I^s \quad (7)$$

To model the capacity constraints we use discrete time slots. For each location  $l \in L$  we define a set  $t \in T$  of time intervals. The set  $C^{l,t}$  contains all tuples  $(s, i)$  of sequence elements in time interval  $t \in T$ . In other words, if the sequence element  $(s, i)$  is associated to a location  $l \in L^{s,i}$ , one capacity unit is needed for this vehicle in time slot  $t \in T$ .

We formulate this capacity constraint as follows:

$$\sum_{(s,i) \in C^{l,t}} \tau_{s,i,l} \leq \sum_{k \in K^l} \sigma_{l,k} \cdot K_{l,k} \quad \forall l \in L, t \in T \quad (8)$$

**Extension** As additional constraint for practical applications, we consider the maximal allowed location utilization over time. Given the duration  $T_t^{Slot}$  for any time slot  $t \in T$  we restrict the average utilization for location  $l \in L$  to a ratio of  $0 \leq U_l^{Max} \leq 1$ :

$$\frac{\sum_{t \in T} \left( T_t^{Slot} \cdot \left( \sum_{(s,i) \in C^{l,t}} \tau_{s,i,l} \right) \right)}{\sum_{t \in T} T_t^{Slot}} \leq U_l^{Max} \cdot \left( \sum_{k \in K^l} \sigma_{l,k} \cdot K_{l,k} \right) \quad \forall l \in L \quad (9)$$

### 4 Complexity

To analyze the complexity of finding an optimal solution for a given model instance, we look at the *Set Covering Problem*, which is a known NP-hard problem (see [4]). Given a zero-one matrix with  $m$  rows and  $n$  columns, we want to cover all rows by choosing a subset of all columns. The number of selected columns should be minimal, that means there is no subset of all columns with smaller size which covers all rows:

$$\min \sum_{i=1}^n x_i \quad (10)$$

$$s.t. \sum_{j=1}^n a_{i,j} x_j \geq 1 \quad \forall i = 1, 2, \dots, m \quad (11)$$

$$x_j \in \{0, 1\} \quad (12)$$

We now transform a given *Set Covering* instance into our model. We set  $L = \{1, 2, \dots, n\}$ , so every column is one potential location. We construct a single sequence  $s$  with one sequence element for any row ( $I^s = \{1, 2, \dots, m\}$ ). We set  $D_{s,i} = 1$  for all  $i \in I^s$ , so that the association to exactly one location (column) is needed for every sequence element.

The set of available locations  $L^{s,i}$  contains all columns which can cover the current row ( $L^{s,i} = \{1 \leq i \leq n \mid a_{i,j} = 1\}$ ). We consider exactly one capacity level for any location and set the costs to  $C_{l,k} = 1$ .

Any optimal solution to our model is also an optimal solution to the covering instance, as there is exactly one column selected for every row with the minimal number of possible locations (columns). Any solution algorithm to our model can also solve any given *Set Covering* instance, therefore our problem is NP-hard to solve.

## 5 Conclusions and Further Research

Our contribution is a new model approach for the cost-optimal selection of charging locations for electric vehicles. Demand is modeled as sequences of vehicle movements and parking activities. Considering the time of parking and a maximum allowed distance, vehicles can be associated to charging locations with a maximum possible increase of range. The objective function is to find a cost-minimal subset of all potential locations. We have shown the problem to be NP-hard and will concentrate on heuristic solution strategies in further research.

## References

1. Luce Brotcorne, Gilbert Laporte, and Frédéric Semet. Ambulance location and relocation models. *European Journal of Operational Research*, 147(3): 451–463, 2003.
2. CC Chan. The state of the art of electric, hybrid, and fuel cell vehicles. *Proceedings of the IEEE*, 95(4): 704–718, 2007.
3. R.Z. Farahani and M. Hekmatfar. *Facility Location: Concepts, Models, Algorithms and Case Studies*. Springer Verlag, 2009.
4. Michael R. Garey and David S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman & Co., New York, NY, USA, 1990.
5. C. Leitinger and G. Brauner. Nachhaltige Energiebereitstellung fuer elektrische Mobilitaet. *e & i Elektrotechnik und Informationstechnik*, 125(11): 387–392, 2008.
6. K. Morrow, D. Karner, and J. Francfort. Plug-in hybrid electric vehicle charging infrastructure review. *Final report Battelle Energy Alliance, US Department of Energy Vehicle Technologies Platform–Advanced Testing Activity*, 2008.
7. H. Neudorfer, A. Binder, and N. Wicker. Analyse von unterschiedlichen Fahrzyklen fuer den Einsatz von Elektrofahrzeugen. *e & i Elektrotechnik und Informationstechnik*, 123(7): 352–360, 2006.
8. S.A. Ross, R. Westerfield, and B.D. Jordan. *Fundamentals of corporate finance*. Tata McGraw-Hill, 2008.
9. D.A. Schilling, V. Jayaraman, and R. Barkhi. A review of covering problems in facility location. *Location Science*, 1(1): 25–55, 1993.
10. Jasna Tomic and Willett Kempton. Using fleets of electric-drive vehicles for grid support. *Journal of Power Sources*, 168(2): 459–468, 2007.
11. Paolo Toth and Daniele Vigo, editors. *The vehicle routing problem*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2001.