Tax neutrality under irreversibility and risk aversion

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Received 20 August 2003; accepted 19 December 2003

Available online 21 March 2004

Abstract

Neutral tax systems that do not affect investment decisions are often considered desirable from a tax policy perspective. This paper uses the real option paradigm to derive neutral tax systems for the first time under risk aversion and irreversibility.

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Keywords: Investment under uncertainty; Real options; Risk aversion; Tax neutrality

JEL classification: H25; H21

1. Introduction

Neutral tax systems that do not affect investment decisions are often considered desirable from a tax policy perspective. Deterministic examples for neutral tax systems are the cash flow tax as derived by Brown (1948) and the taxation of true economic profits (e.g. Samuelson, 1964; Johansson, 1969). In recent years, public economists were especially interested in tax neutrality under uncertainty. Thus, real option literature based on the Dixit and Pindyck (1994) paradigm has been enriched by taxation (e.g. Harchaoui and Lasserre, 1996; Agliardi, 2001). By doing so, it is possible to derive investment rules considering managerial flexibility, irreversibility, taxational effects, and to identify tax systems that do not distort investment decisions. For risk neutrality, such neutral tax systems have already been proved in the real option context (e.g. Niemann, 1999; Sureth, 2002).
This paper extends the model design to risk aversion. We succeed in deriving neutral tax systems, but in contrast to the pre-tax case, real option theory is not able to provide general solutions for investment decisions after taxes. Integrating even simple tax systems reveals severe complications under risk aversion. This problem is caused mainly by taxation of interest income, inducing different discount rates before and after taxes.

2. Model setup

In accordance with the Dixit and Pindyck (1994) model, we consider an option to invest in a project with stochastic cash flows. Its owner faces the decision between either exercising the option, which means stopping waiting, carrying out the project and collecting the resulting cash flow or continuing waiting and sacrificing cash flows but keeping the option to avoid unexpectedly low cash flows. Given the value of the investment project, which is the underlying asset, the value of the option to invest can be determined. Since the owner of the option can only decide between waiting and exercising, the decision variable is binary, and it is possible to determine the optimal decision by complete enumeration.

Applying dynamic programming, we will start with the continuation region in which the option is kept alive. The optimal transition to the exercise region will be modeled by boundary conditions.

We assume a simple tax system and neglect further aspects of real-world tax systems. The profit tax base equals cash flow \( p \) less depreciation allowances \( d \) that may be deterministic or stochastic. The tax rate \( s \) is assumed deterministic and constant. Under an immediate loss-offset, the post-tax cash flow \( ps \) is defined as

\[
ps = \frac{1}{1 + sc} p + sd
\]

Cash flows are a function of the geometric Brownian motion \( P \) (with \( \frac{dP}{P} = \alpha dt + \sigma dz \), where \( \alpha \) and \( \sigma \) denote the growth and volatility parameter, respectively) and time \( t \):

\[ \pi = \pi(P, t) \]

As long as the option to invest is not exercised, available funds yield the risk-free constant capital market rate \( r \). \( \gamma \) denotes the taxable fraction of interest income. The risk-free after-tax interest rate \( rs \) can be written as

\[
rs = \left( \frac{1}{1 + c} \right) r
\]

The project’s economic life \( T \) is finite, cash flows equal zero after time \( T \): \( \pi(P, T) = 0 \). Further, a fraction \( D_I \) of the project’s initial outlay \( I_0 \) might be written off immediately. To separate immediate write-offs from current depreciation deductions, the former are included in the effective initial outlay \( I_0^{\text{eff}} \), where \( I_0 \) is reduced by the tax shield on the immediate write-off: \( I_0^{\text{eff}} = (1 - Di) I_0 \).

The option to invest is assumed depreciable, too. Depreciation deductions associated with the option to invest are denoted by \( d_F \in \mathbb{R} \). Accordingly, the option might involve a non-zero cash flow \( \pi_F = \tau d_F \). Both depreciation allowances \( d_V \) and \( d_F \) may be stochastic as well as deterministic.

The investor’s risk preferences are endogenized by his utility function \( U(\pi) \): \( U \) is assumed time-invariant (\( \frac{\partial \pi}{\partial t} = 0 \)), twice continuously differentiable, with positive and diminishing marginal utility (\( (\partial^2 U/\partial \pi^2) < 0 \)) and additive over time. Without restricting generality, utility is standardized to \( U(0) = 0, U(1) = 1 \). Disutility from an investment project’s initial outlay \( U(-I_0) < 0 \) is treated as an individual constant. The investor’s time-preference parameter \( \rho \) used as the discount rate is assumed constant.

To permit tax-induced changes in the individual time-preference rate the post-tax discount rate is defined as \( \rho_s = (1 - \gamma \tau) \rho \), where \( \gamma \) denotes an exogeneously-given individual parameter measuring the

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1 Variables concerning the option are denoted with subscript \( F \), variables concerning the investment project with subscript \( V \).
response of the investor’s time-preference rate to the tax system. $\psi$ does not necessarily correspond to the tax parameter $\gamma$.

3. Risk aversion and taxes

An investment project already in place does not offer any flexibility, so its value simply consists of its expected discounted utility. The project’s pre-tax value $V$ is given by:

$$V(P, t) = E\left[\int_t^T U(\pi_V(P, \xi))e^{-\rho(\xi-t)}d\xi\right] = \int_t^T E[U(\pi_V(P, \xi))]e^{-\rho(\xi-t)}d\xi,$$

the after-tax project value by:

$$V_\tau = \int_t^T E[U[(1-\tau)\pi_V + \tau d_V]]e^{-\rho_\tau(\xi-t)}d\xi.$$  

Exercising the option to invest is optimal when the investment project’s discounted future utility exceeds the initial outlay’s utility plus the option’s utility. Determining the value of this option requires the following continuous-time Hamilton–Jacobi–Bellman equation:

$$\rho_\tau F_\tau = U(\pi_F) + \frac{E[dF_\tau]}{dt}. \quad (3)$$

Its economic interpretation is that the owner of the option expects an instantaneous return that in equilibrium equals the post-tax discount rate. The post-tax partial differential equation for the option value is

$$\frac{\partial F_\tau}{\partial t} + \frac{1}{2} \sigma^2 P^2 \frac{\partial^2 F_\tau}{\partial P^2} + \rho_\tau \frac{\partial F_\tau}{\partial P} - \rho_\tau F_\tau + U(\pi_F) = 0 \quad (4)$$

with the fixed boundary conditions

$$F_\tau(0, t) = \int_t^T E[U(\pi_F(0))]e^{-\rho_\tau(\xi-t)}d\xi, \quad F_\tau(P, T) = 0 \quad (5)$$

and the free boundary conditions

$$F_\tau(P^*, t) = \frac{1}{2} \frac{\partial V_\tau(P^*, t)}{\partial P} + U(\pi_F(0))$$

$$= \frac{1}{2} \frac{\partial V_\tau(P^*, t)}{\partial P}. \quad (6)$$

Eq. (5) implies that a call on a worthless underlying is itself worthless. Eqs. (6) and (7) are free boundary conditions determining the transition from the continuation region to the exercise region at the critical investment threshold $P^*$. The so-called value-matching condition (6) means that a project’s benefits must equal its costs at the point of transition. Eq. (7) is called smooth-pasting or high contact condition requiring the identity of marginal benefits and marginal costs at the critical threshold.
Obviously, intertemporal models with differing pre- and after-tax discount rates are quite complicated. As can be seen from Eqs. (1) and (2) a neutral tax system has to balance a tax base as well as a discounting effect.

Since both the pre- and post-tax problems involve free boundary problems with partial differential equations, they typically cannot be solved analytically. I.e. it is not possible to derive the critical investment threshold \( P^*_s \) explicitly. As a consequence, neutral tax systems cannot be derived by simply equating the pre-tax and the post-tax investment thresholds, and herewith it is not possible to refer to the conventional neutrality condition that is necessary and sufficient.

Nevertheless, we can fall back on a condition that is only sufficient. Using the pre-tax model as a yardstick for measuring tax effects, it is possible to derive neutral tax systems by comparing the pre-tax and post-tax problems rather than their solutions. Identical solutions may be achieved by different problems whereas identical problems necessarily yield identical solutions. The investment problems under consideration are identical if the pre-tax and post-tax partial differential equations and boundary conditions are equivalent.

The value matching and smooth pasting conditions before and after taxes are equivalent if all post-tax values—measured in utility units—are a multiple \( c \) of their pre-tax counterparts. The resulting neutrality conditions can be summarized as follows. Proportionality of disutility from the initial outlay before taxes \( U(-I_0) \) and after taxes \( U(-I^\text{eff}_0) \) can be achieved by granting a utility-dependent immediate write-off:

\[
D_i = \frac{1}{\tau} + \frac{U^{-1}[c U(-I_0)]}{\tau I_0} \Rightarrow U(-I^\text{eff}_0) = c \cdot U(-I_0). \tag{8}
\]

As far as the proportionality of pre-tax and after-tax project values is concerned, the non-separability of cash flow- and depreciation-related utility components does not allow computing neutral depreciation deductions in present value terms. Thus, it is necessary to derive a neutral depreciation schedule:

\[
d_V = \frac{U^{-1}[c U(\pi_V) - \psi \tau \rho c V]}{\tau} - \frac{1 - \tau}{\tau} \pi_V \Rightarrow V_t = c \cdot V. \tag{9}
\]

Additionally, a depreciation schedule for the option to invest that implies the proportionality of the option values before and after taxes is given by:

\[
d_F = \frac{1}{\tau} U^{-1}(-\psi \tau \rho F_t) = \frac{1}{\tau} U^{-1}(-\psi \tau \rho c F) \Rightarrow F_t = c \cdot F. \tag{10}
\]

Here, it becomes obvious that depreciation allowances on the option to invest are necessary to meet the sufficient neutrality condition—at least if taxation has an impact on the time preference rates. The neutral depreciation schedules for the investment project and the option to invest are preference-dependent. Typically, it is not possible to eliminate the utility function \( U \) and its inverse \( U^{-1} \) from \( d_i \), \( d_F \) and \( D_i \). Since different investors are characterized by different utility functions, neutral taxation cannot refer to an objective tax base.
4. Dynamic programming versus contingent claims analysis

A real option-based investment model requires the decision between two approaches: either dynamic programming, as used above, or contingent claims analysis, a no-arbitrage pricing approach derived from financial option theory (cf. Black and Scholes, 1973). Both methods yield equivalent results in the tax-free case. This raises the question if there is a superior approach. Dynamic programming, if interpreted as an individual approach, neglects information from market data. For example, it is not possible to distinguish between unsystematic and systematic risk. The risk-adjusted discount rate to be used in contingent claims analysis has to be derived from capital market equilibrium. Unfortunately, a dynamic after-tax CAPM is not yet available. Thus, the general derivation of neutral tax systems cannot rely on contingent claims analysis.\(^2\)

5. Conclusion

We proved neutral tax systems under uncertainty and risk aversion. Tax neutrality in a real option context under risk aversion requires three conditions: a neutral depreciation schedule for the investment project, a neutral depreciation schedule for the option to invest, and a utility-dependent immediate write-off of the initial outlay. The conditions’ complexity is mainly caused by the taxation of interest income, which induces different discount rates before and after taxes. It is possible to apply the neutrality conditions derived here to any investor with a given utility function. Moreover, our model can be easily extended to investment situations with partial reversibility, e.g. multi-stage projects like R&D. As a consequence, real option-based models are preferable to models that do not include the effects of irreversibility. If future research in Finance will succeed in developing a dynamic post-tax capital market equilibrium model, contingent claims analysis may lead to even more general results.

References


\(^2\) In contrast to dynamic programming, contingent claims analysis requires the so-called spanning property which further restricts the applicability.