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Is there more voluntary disclosure if investors are better informed?
Is there more voluntary disclosure if investors are better informed?

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Abstract

This paper studies the interaction of investor’s knowledge about a manager’s information endowment and their ability to gauge the information content on voluntary disclosure. If investors are unable to discern anything beyond the manager’s information endowment, a higher probability of informed investors increases voluntary disclosure. However, if investors are able to discern the manager’s private information and not just her information endowment the result flips—nondisclosure becomes more prevalent as investors are informed with a higher probability. These results have implications for empirical researchers as well as regulators: The incentives for disclosure provided by rational expectations are very sensitive to the investors’ sophistication in a given market.

Key Words: Information endowment, Voluntary disclosure, Real effects

JEL classification: M41, D82, C02

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1 Introduction

The purpose of this paper is to study the effect of two-sided uncertainty on voluntary disclosure. We study voluntary disclosure by firms that chance upon private information regarding their own value and which are uncertain about whether the capital market knows about their information endowment. We ask the following simple question: Do better informed investors imply more or less voluntary disclosure? It turns out that this question has no simple answer. We show that learning about the information endowment, i.e. the fact that the firm has private information, interacts with the ability to gauge the information content.

A variation of this question has been addressed by Dye (1998), who derives the result that voluntary disclosure increases if the proportion of sophisticated investors in the market, i.e. those that are able to observe the firm’s information endowment, increases. We show that this result is sensitive to the type of sophistication. If investors observe the information itself together with the information endowment, we show that the result flips – voluntary disclosure decreases.

We derive our results by considering two sets of assumptions. The first set resembles Dye (1998) in that investors, conditionally on the firm having private information, may learn about the firm’s information endowment but not about the information content. If they do and the firm does not disclose, the investors price it at its expected value conditional on the information being sufficiently bad – a low nondisclosure price. In all other cases of nondisclosure the investors consider the possibility that the firm is indeed uninformed and trade it at a higher nondisclosure price. From the firm’s perspective the probability with which investors observe its information endowment determines how valuable nondisclosure is. If this probability increases, it becomes less likely that the firm commands the high nondisclosure price. This effect encourages voluntary disclosure. At the same time the high nondisclosure price itself increases because investors deem it
conditionally more likely that the firm is uninformed. This effect discourages voluntary disclosure.

Our second set of assumptions is different in but one respect: If investors observe the information endowment of a firm, we assume now that they are also able to gauge the information content and price the firm accordingly. Thus, from the firm’s perspective nondisclosure becomes more desirable, because if it is caught withholding information, it gets priced at its actual value and not a low expected value. Put differently, since low expectations in case of deliberate withholding are replaced by the actual firm value, the firm is indifferent between disclosure and getting caught withholding information. Thus, rational expectations lose a lot of their power to unravel nondisclosure.

In addition to Dye (1998) our paper relates to previous literature on voluntary disclosure in several ways. Jung and Kwon (1988) discuss an extension of voluntary disclosure with uncertain information endowment of the manager, were the manager fears that investors obtain an unfavourable signal about firm value from an outside source. They show that this helps unravel nondisclosure because the manager has an incentive to release all information which is marginally better than the outside signal. Our second set of assumptions can be understood as a world where investors, upon learning that the firm has private information, are able to uncover this information through the use of outside sources such as analysts, news agencies, social media or the like.1

In our model the informed manager is uncertain about how investors are going to price the firm in case she opts for nondisclosure. This is because she does not know whether investors are aware of her information endowment or not. In this respect our paper relates to Suijs (2007). There the firm is uncertain about investors’ reaction to disclosure because the latter have private information about an outside investment opportunity and

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1For empirical evidence regarding the role of these information sources see for example Fang and Peress (2009), Bushee et al. (2010), Tetlock (2010), Li, Ramesh, and Shen (2011), Griffin, Hirschey, and Kelly (2011), and C.-W. Chen, Pantzalis, and Park (2013) or Saxton (2012), Saxton and Anker (2013), Hu et al. (2013), and H. Chen et al. (2014).
benchmark the firm’s disclosures against this information. Together both models help us understand how managers might deal with uncertain market reactions to their disclosure decisions.

The paper proceeds as follows. In section 2 we introduce the model and specify payoffs under two different sets of assumptions. In section 3 we derive our base results and relate them to Dye (1998). In section 4 we derive results under the competing set of assumptions and in section 5 we compare results and derive empirical implications. Section 6 concludes the paper.

2 The Model

Consider a single-period model of a firm with uncertain terminal value $\tilde{x}$. At the beginning of the period risk-neutral investors and the firm’s manager share prior beliefs about $\tilde{x}$, represented by a probability function $F$ with support $[x, \bar{x}]$ and mean $\mu$. It is common knowledge that with probability $(1 - p)$ the manager privately observes the true firm value $x$ at the beginning of the period and with probability $p$ she observes nothing. Formally, the manager has private information $\Omega_M \in \{x, \emptyset\}$, where $\emptyset$ indicates a lack of private information. Upon observing $x$ the manager is able to disclose it credibly and without cost so that the firm’s market price $P$ after disclosure becomes $x$. She is unable, however, to make a credible disclosure that she did not observe $x$. Hence, her action space $d$, conditional on observing $x$, is $d|(\Omega_M = x) \in \{x, ND\}$, where $ND$ denotes nondisclosure. If she does not receive a private signal her action space reduces to $d|(\Omega_M = \emptyset) \in \{ND\}$. In order to abstract from potential agency related problems we assume, as Dye (1985) and Jung and Kwon (1988) do, that the firm’s current shareholders agree to a disclosure policy which maximizes the firm’s market price and that the manager adopts this policy.

If the manager has private information, we consider two scenarios.
First scenario: The investors observe the manager’s information endowment with probability \((1 - q)\).\(^2\) Hence, the manager is aware that if she observes \(x\) and does not disclose it, there is a chance that investors know she deliberately held back information. Formally, after observing disclosure or nondisclosure the investors have private information \(\Omega_I \in \{\{x\}, \{\square x, ND\}, \{\Diamond x, ND\}\}\), where \(\square x\) symbolizes that investors know the manager observed \(x\), and \(\Diamond x\) means the investors think it possible that the manager observed \(x\).\(^3\)

Second scenario: The investors observe the manager’s information endowment with probability \((1 - q)\). If they do, they are able to discern \(x\). Formally, after observing disclosure or nondisclosure the investors have private information \(\Omega_I \in \{\{x\}, \{x, ND\}, \{\Diamond x, ND\}\}\). Note that the only difference to the first scenario is that the set \(\{\square x, ND\}\) is replaced by the set \(\{x, ND\}\). The information structure in both scenarios is common knowledge to all players.

Investors price the firm based on all available information. Hence their action space is \(P(\tilde{x}|\Omega_I)\). Note that \((1 - q) > 0\) introduces uncertainty of the manager about the information endowment of investors. In other words, the manager is uncertain about how investors’ reaction to nondisclosure.

Figure 1 depicts the complete game. Investors perceive ex ante that one of four mutually exclusive events, denoted A, B, C, D will be realized. Figure 1 shows that the two scenarios only differ in event B where investors know that the manager deliberately retained private information. In the first scenario this is all the investors know, i.e. \(\Omega_I = \square x\), in the second scenario investors also know exactly what information the manager has, i.e. \(\Omega_I = x\).

\(^2\)By assumption the investors do not wrongly suspect the manager of having private information when in fact she does not.
\(^3\)Notation stems from modal logic where \(\square\) is an operator meaning Necessarily and \(\Diamond\) is an operator meaning Possibly.
Figure 1: Model overview
Event A is that the manager observes and discloses $x$. In event C the manager withholding $x$ unbeknownst to the investors. And finally, event D is that no information is received by the manager. Rational expectations under the non-disclosure events are as follows.

Upon disclosure in event A, rational expectations take the value of the disclosed signal $x$ and investors price the firm at $P(\tilde{x}|x) = E[\tilde{x}|A] = x$.

Event D is a no information event. Hence, if rational investors believe the manager received no private information, their posterior expectations equal prior expectations, i.e.:

$$E[\tilde{x}|D] = \mu \quad (1)$$

If investors believe that event C has occurred, they assume that the manager must have observed an unfavorable realization $x$ in the sense that it is weakly below some threshold $y$ with $y \in [\underline{x}, \mu]$. Let $E_y$ be the conditional expectation of $\tilde{x}$ given event C with

$$E_y = E[\tilde{x}|x \leq y] = \int_{\underline{x}}^{y} \frac{x f(x)}{F_y} dx, \quad (2)$$

and

$$F_y = \text{Prob}(\tilde{x} \leq y) = \int_{\underline{x}}^{y} f(x) dx.$$ 

Intuitively, any non-disclosure interval must be bounded above by a threshold $y$. To see this, consider a manager who observes $x = \bar{x}$. Her optimal choice is disclosure. The same is true for a manager who observes a slightly lower value $x = \bar{x} - \varepsilon$ because she cannot pool with $\bar{x}$ and does not want to pool with anybody else. This argument continues until pooling with uninformed managers becomes optimal although it inevitably entails pooling with worse (informed) firms.

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4Note that the manager pursues a strategy of price maximization. Since we assume her disclosures to be truthful and costless, she has no incentive to be silent about any $x > \mu$. 

7
Investors cannot distinguish between events C and D because in both cases they do not observe the manager’s information endowment. Their posterior expectations given $\Omega_I = \{\hat{x}, ND\}$ are:

$$\text{Prob}(C|\hat{x}, ND) = \frac{(1 - p)qF_y}{p + (1 - p)qF_y}$$

(3)

and

$$\text{Prob}(D|\hat{x}, ND) = \frac{p}{p + (1 - p)qF_y}.$$  

(4)

The resulting market price is a linear combination of rational expectations for both events.

$$P(\hat{x}|\hat{x}) = \frac{p}{p + (1 - p)qF_y} \cdot \mu + \frac{(1 - p)qF_y}{p + (1 - p)qF_y} \cdot E_y$$

(5)

Pricing as described above is identical in both scenarios. In event B, however, pricing differs between the two scenarios. In the first scenario the investors know in event B that the manager retains information. Thus, they conclude that it must be sufficiently bad and the market price based on rational expectations is

$$P(\hat{x}|\square x) = E_y.$$  

(6)

This is different in the second scenario, where investors in event B know exactly what information the manager tries to hide, i.e. they observe $x$. Hence they price the firm at $x$ instead of $E_y$.

3 First scenario: Information endowment

In this section we investigate the first scenario and link it to the results derived in Dye (1998). In addition we identify two countervailing forces which drive the manager’s disclosure decision. These two forces are key to understanding why our two scenarios produce different results. Remember that in the first scenario in event B the investors’
information is □x. Then the set \( \mathcal{N} \) of realizations, which the manager would not disclose, is given by:

\[
\mathcal{N} = \{ x | (1 - q)P(\tilde{x} | \square x) + qP(\tilde{x} | \Diamond x) \geq x \}
\]  

(7)

In a rational expectations equilibrium the conjectures of investors with respect to the disclosure threshold \( y \) need to be fulfilled. Likewise the manager’s expectations regarding investors’ reaction to non-disclosure need to be true in equilibrium. It follows that:

\[
y = \sup \mathcal{N} = (1 - q)P(\tilde{x} | \square x) + qP(\tilde{x} | \Diamond x)
\]  

(8)

In the first part of the proof to Proposition 1 we show that there exists a disclosure threshold \( y \), which is a function of \( q \) (or equivalently \( 1 - q \)). A closer inspection of the mechanics underlying the relation between \( y \) and \( (1 - q) \) reveals two opposing effects. In particular, an increase in \( (1 - q) \) makes it less likely that the manager successfully hides the existence of private information. Accordingly, if the manager withholds information, she expects the market to pay the low nondisclosure price \( P(\tilde{x} | \square x) \) with a higher probability. On the other hand, if investors do not observe a signal about the manager’s information endowment, they deem it (conditionally) more likely that the manager is indeed uninformed if \( (1 - q) \) increases. To see this, consider the conditional probability that the manager deliberately hides \( x \), given that investors have no private signal and observe nondisclosure by the manager:

\[
Prob(\Omega_M = x | ND, \Omega_I = \Diamond x) = \frac{(1 - p)qF_y}{p + (1 - p)qF_y}
\]

\( Prob(\Omega_M = x | ND, \Omega_I = \Diamond x) \) is the probability investors assign to event C, conditional on them observing nondisclosure and being oblivious of the manager’s information en-
dowment. The derivative with respect to \( q \) of this conditional probability is:

\[
\frac{\partial}{\partial q} \left( \frac{(1 - p)qF_y}{p + (1 - p)qF_y} \right) = \frac{p(1 - p)F_y}{(p + (1 - p)qF_y)^2}.
\]

(9)

Thus, equation (9) shows that an increase in \( q \) (a decrease in \( (1 - q) \)) increases the weight assigned to event C, which yields an expected value of \( E_y \). Vice versa this means that an increase in \( (1 - q) \) decreases the weight assigned to event C and increases the weight on the uninformed state (event D), which yields expectations \( \mu \). Hence, successful nondisclosure yields a higher market price \( P(\tilde{x}|\Diamond x) \) the higher \( (1 - q) \) is. Corollary 1 captures this result.

**Corollary 1.** As investors on average get more informed about the manager’s information endowment we observe the following two countervailing effects:

1. The manager’s benefit from withholding unfavourable information increases, if it remains undetected – \( P(\tilde{x}|\Diamond x) \) increases in \( (1 - q) \).

2. The investors detect more often that the manager withholds information. If \( (1 - q) \) increases the manager is less often able to command a price of \( P(\tilde{x}|\Diamond x) \). Instead she receives the lower price \( P(\tilde{x}|\Box x) \).

Which of the above effects dominates? Proposition 1 shows that the latter effect described in Corollary 1 dominates the former for a large set of assumptions regarding \( F \). Therefore the manager discloses more often, if she believes investors to be better informed about her information endowment.

**Proposition 1.** Assume that the investors obtain information about the manager’s information endowment. Then the disclosure threshold \( y \) is a decreasing function of \( (1 - q) \) for a large set of distributions \( F \) (proof in the appendix).

The comparative statics of the above proposition confirm the results in Dye (1998), i.e. better informed investors imply more disclosure. This result will be a benchmark for our
further analysis. Remember that according to Corollary 1 there are two countervailing forces at play. However, as the above proposition has shown, the one that decreases the disclosure threshold, if investors become better informed, is always stronger. The following section discusses the second scenario. Here, if investors learn about the information endowment of the manager, they also obtain the manager’s private information itself. We are going to show that the comparative statics with respect to changes in $(1 - q)$ flip as compared to the first model.

4 Second Scenario: Information content

In this scenario the investors, conditional on observing the manager’s information endowment, also observe the manager’s private information itself. Thus, with probability $(1 - q)$ they observe $x$ rather than $\Box x$. It appears that now in event B the investors have more information than in the first scenario ($x$ is a better statistic about $x$ than $\Box x$). In all other states (A, C and D) nothing changes. This seemingly small change to the information structure has a tremendous effect on our results. We show in the remainder of this section that the comparative static results flip entirely. This is especially interesting because it extends our understanding of the role which external information providers play in firms’ disclosure policies. We discuss this issue further after deriving the results.

In order to symbolize that we are in the setting in which investors learn the actual content of the manager’s private information we use superscript $x$ on the equilibrium. In particular, we denote the nondisclosure set by $\mathcal{N}^x$ (and the corresponding disclosure threshold by $y^x$). Remember that investors, upon learning $x$, price the firm at $x$. Accordingly, the nondisclosure set $\mathcal{N}^x$ must satisfy

$$\mathcal{N}^x = \{x | (1 - q)x + qP(\tilde{x}|\Diamond x) \geq x\}$$

(10)
The condition on the set,

\[(1 - q)x + qP(\tilde{x}|\diamond x) \geq x\]

is equivalent to

\[P(\tilde{x}|\diamond x) \geq x.\]

Independent of the form of the set \(\mathcal{N}^x\) the price \(P(\tilde{x}|\diamond x)\) is a constant. Since the right-hand side of the inequality increases in \(x\) there exists a value \(y^x\) such that \(\mathcal{N}^x = [x, y^x]\).

Rational expectations require that the conjectures of investors with respect to the disclosure threshold \(y^x\) are correct in equilibrium. Likewise the manager’s expectations regarding investors’ reaction to nondisclosure need to be true in equilibrium. It follows that

\[y^x = \sup \mathcal{N}^x = (1 - q)x + qP(\tilde{x}|\diamond x) \quad (11)\]

or \(y^x = P(\tilde{x}|\diamond x)\). For unspecified distributions \(F\) the equilibrium disclosure threshold \(y^x\) is implicitly defined by (11).

**Proposition 2.** Assume that the investors privately observe the manager’s private information with probability \((1 - q)\). Then the disclosure threshold \(y^x\) is an increasing function of \((1 - q)\) for a large set of distributions \(F\) (proof in the appendix). Thus, if investors become better informed, the level of disclosure decreases.

Proposition 2 yields the exact opposite result to Proposition 1. Remember that under the assumptions of Proposition 1 better informed investors imply more disclosure, and we found this to be in line with Dye (1998). In order to understand why Proposition 2 finds opposite results it is helpful to revisit the two countervailing effects identified in Corollary 1.

From Figure 1 recall that, if the manager has private information, depending on her disclosure decision, she faces one of three potential outcomes:
(i) The manager discloses her information, in which case she reveals her information endowment.

(ii) The manager withholds her information and investors are oblivious about her information endowment (successful nondisclosure).

(iii) The manager withholds her information and investors know that she does (unsuccesful nondisclosure).

In case (i) the manager realizes a market value of $x$, which is no different from what she would get in the first scenario. In case (ii) the investors do not know whether the manager has or has not obtained private information. Therefore, as in the first scenario, the investors average between a firm that has no private information and a firm that has but does not disclose it, i.e. a firm in the set $[\underline{x}, \overline{y}]$. Therefore, the investors structurally price the firm exactly as before: at $P(\hat{x} \mid \hat{x} x)$. All else equal, the price will differ if compared to the first scenario only because the disclosure thresholds are different: $\overline{y} \neq y$. The scenarios differ considerably, however, if it comes to case (iii). Under the first scenario, all that investors learned was that the manager withheld information. Rational expectations implied that the information must have been sufficiently bad, which lead to a price of $E_y$. Thus, being caught not disclosing private information potentially came at a price to the manager. If her private signal exceeded $E_y$ she would have preferred disclosure ex post (but not ex ante, because ex ante she compared $x$ with a weighted average of $E_y$ and $\mu$). By the same argument she would have preferred nondisclosure ex post if $x < E_y$.\(^\text{5}\) This potential cost to or gain from nondisclosure is not present in the second scenario. If the manager expects investors to know $x$ and price the firm accordingly, there are no ex post regrets to nondisclosure. In other words, if investors are able to gather value relevant information that the manager has from sources

\(^{5}\text{Note that under the assumptions in the first scenario a manager with type } x < E_y \text{ gains from nondisclosure, even is she is caught withholding information.}\)
other than the manager, nondisclosure does not necessarily imply pessimistic beliefs. To summarize: in our second scenario there is no threat of being punished for not disclosing. Therefore the only thing the manager cares about when making his disclosure decision is whether the price she obtains from uninformed investors (case (ii); $P(\tilde{x}|\Diamond x)$) is above or below her true type. This is exactly what the reformulated equilibrium condition $y^x = P(\tilde{x}|\Diamond x)$ reflects.

Under the first scenario Corollary 1 identified two countervailing effects. One of these effects always dominated: Increasing $(1 - q)$ lowers the expected nondisclosure price because the manager expects to command the high nondisclosure price ($P(\tilde{x}|\Diamond x)$) with a lower probability. We showed above that this effect is absent from the second scenario, although the manager still associates an increase in $(1 - q)$ with a lower probability of realizing $P(\tilde{x}|\Diamond x)$. However, the price she expects to receive instead is $x$, which makes deliberate nondisclosure lose part of its downside. Of course the other effect is still very much present: if investors are oblivious of the managers information endowment, they still rationally associate nondisclosure with low realizations of $x$. An increase in $(1 - q)$ lowers the weight investors put on this possibility and the nondisclosure price $P(\tilde{x}|\Diamond x)$ increases. Since this is the sole effect of an increase in the investors informedness (or sophistication) it dominates in the second scenario: As investors become informed with a higher probability, firms forego voluntary disclosure more often.

5 Discussion and empirical implications

In this section we compare the two scenarios - knowledge about the information endowment and knowledge of the actual information - with reference to a numerical example. On the one hand this helps to gain some intuition and on the other hand it nicely visualizes the differences of Propositions 1 and 2. Figure 2 assumes $p = 0.3$ and that $x$
is drawn from a continuous uniform distribution with support $[1, 2]$, i.e. $\tilde{x} \sim U[1, 2]$. Note that, given these assumptions, the requirements on the distribution function $F(\cdot)$ in Propositions 1 and 2 are satisfied.

Figure 2 plots the disclosure thresholds $y$ and $y^x$ as functions of $q$. The dashed curve represents the disclosure threshold when investors might learn the manager’s private information (second scenario). The solid curve represents the disclosure threshold when investors might learn about the manager’s information endowment alone (first scenario). In both cases higher values of $q$ represent less informed investors. In accordance to Propositions 1 and 2 the solid curve is increasing in $q$ (this is consistent with the results in Dye (1998)) and the dashed curve is decreasing in $q$. Therefore, investors being informed with a higher probability implies more disclosure in the first scenario and vice versa in the second scenario.

The two functions coincide for $q = 1$. This is not driven by our choice of parameters but generally true. To see this, remember that $q = 1$ corresponds to the case in which the investors never gain knowledge of the manager’s information endowment (or the information content) and hence the difference between both scenarios ceases to matter. Figure 2 also shows that the disclosure threshold $y^x$, i.e. when investors potentially uncover the manager’s private information itself, is always (for each value of $q$) above the disclosure threshold in case that investors merely observe the information endowment, $y$. As investors are better informed if they learn about the manager’s actual information rather than only about her information endowment, our results show that better informed investors imply less disclosure.

A general proof for this claim follows directly from the results above. It has been established that the curves representing the two settings coincide for $q = 1$. The function $y^x(q)$ is decreasing in $q$ (see Proposition 2) and the function $y(q)$ is increasing in $q$.
(Proposition 1). Hence, it follows that for all $q \in [0, 1]$

$$y^x(q) > y^x(1) = y(1) > y(q) \iff y^x(q) \geq y(q)$$

Figure 2: Disclosure thresholds $y$ and $y^x$ as a function of $q$ for $p = 0.3$ and $\hat{x} \sim U[1, 2]$

The following proposition summarizes the result.

**Proposition 3.** For fixed values of $q$ the disclosure threshold in case of knowledge about the actual information is always (for each value of $q$) above the threshold in case of knowledge about the information endowment. Thus, if investors are better informed the level of disclosure decreases.

An intuitive explanation for the result in Proposition 3 is the following. Remember that the manager at the margin (that is the one with the highest private signal among all nondisclosers) determines the level of disclosure. In our first scenario (information
endowment is observed) the manager has something to lose from nondisclosure. If she is
cought not disclosing her private signal she realizes a market price, which is below firm
value: $y > E_y$. This is not the case in the second scenario, where investors potentially
uncover the manager’s actual private signal. Here, in the same situation (being caught
not disclosing) the manager realizes exactly the same market price as in case of voluntary
disclosure, i.e. $y$. Therefore the manager with type $y$ has strictly higher incentives not
to disclose in our second setting, which leads to less disclosure in equilibrium.

Note that Proposition 3 is conservative with respect to the implications of our analysis.
For example it is plausible to imagine that the values for $(1 - q)$ differ between the
scenarios, because for the investors it seems easier to obtain information about the
manager’s information endowment than about her actual private signal. However, even
if the probabilities in the two settings differ, it will always be true that there is more
disclosure in the former case than in the latter.

Our results have implications for empirical research. Most notably, the information
available to investors from third parties is likely to have a nontrivial effect on voluntary
disclosure, depending on how informative it is about the firms’ private information. Vari-
ations in informativeness could either be rooted in the quality or number of externally
available information sources, or in investor sophistication. For instance: When the in-
vestors’ private signal is limited to be very uninformative (for example because it is just
a random signal about the manager’s information endowment) more external informa-
tion sources should lead to more voluntary disclosure by firms because they increase the
probability that investors actually observe the manager’s information endowment. At
the same time, for a given number of external information sources, voluntary disclosure
is going to be affected positively, if the informativeness of the external information de-
creases and negatively affected, if it increases. Therefore, empirical research on voluntary
disclosure by firms needs to capture the information environment on at least these two
dimensions: (i) the probability that external information is available to investors, and (ii) the information content of that information (or alternatively investor sophistication in understanding the available information).

6 Conclusion

In this paper we presented a model that took its inspiration from Dye (1985) and Dye (1998). As Dye (1998) does, we assume that investors stochastically and conditional on the firm having information, observe a private signal about either the firms information endowment alone, or its endowment and the information content. However, unlike Dye (1998), who is concerned with the relation between the proportion of informed investors and voluntary disclosure, we show how voluntary disclosure depends on the interaction of the investors likelihood to learn about the firm’s information endowment with the extent of that learning. As a base result we show that voluntary disclosure increases, if firms think it becomes more likely that investors have knowledge about their information endowment. Although our model differs significantly from Dye (1998), this result follows from the same economic intuition. In a slightly changed setting we show voluntary disclosure decreases, if firms think it becomes more likely that investors have knowledge about their information endowment and the actual information content. This result is surprising, because it implies that firms might be able to externalize parts of their voluntary disclosure activities. In an extreme interpretation this could mean that firms, which are under high scrutiny by analysts, the press or social media, minimize their disclosures because these outside parties supply investors with all relevant information. We leave it to empiricists to analyze, if and under what circumstances firms exhibit such a behavior.
Proofs

Proof of Proposition 1. First we show that there always exists a non-trivial disclosure policy characterized by a disclosure threshold $y$ with $x < y < \mu$, such that the manager withholds all private signals $x \leq y$.

The proof is similar to that of proposition 1 in Jung and Kwon (1988). Substituting (5) and (6) into (8) and rearranging terms yields:

$$p\mu - \frac{p}{q}(y - (1 - q)E_y) = (1 - p)F_y(y - E_y)$$ \hspace{1cm} (12)

Now let $y = x$. Then the left-hand side (LHS) of (12) simplifies to $p(\mu - x) > 0$ and the right-hand side (RHS) of (12) simplifies to 0. Thus for $y = x$ the LHS is strictly larger than the RHS. Secondly, let $y = \mu$. Then the LHS of (12) simplifies to

$$\frac{p(1 - q)}{q}(E_\mu - \mu) < 0$$ \hspace{1cm} (13)

and the RHS of (12) simplifies to

$$(1 - p)F_\mu(\mu - E_\mu) > 0$$ \hspace{1cm} (14)

Accordingly, for $y = \mu$ the LHS is strictly below the RHS. Since both the LHS and the RHS are continuous in $y$ for all continuous distributions $F$, there always exists a value of $y$ satisfying (12) such that $x < y < \mu$.

Secondly, we show that $y$ decreases in $(1 - q)$. The non-disclosure set $\mathcal{N}$ as defined in (7) contains all private observations $x$ that yield an expected non-disclosure price higher
than $x$. Thus, the equilibrium condition is

\[ y(q) = P_{\text{ND}}(y(q), q) \]

where

\[ P_{\text{ND}}(y(q), q) = (1 - q)P(\tilde{x} | \square x) + qP(\tilde{x} | \diamondsuit x) \]

\[ = (1 - q)E_y + q \left[ \frac{p}{p + (1 - p)qF_y} \mu + \frac{(1 - p)qF_y}{p + (1 - p)qF_y} E_y \right] \]

denotes the expected non-disclosure price. For brevity of notation, in what follows we drop the arguments from the expected non-disclosure price, i.e. we let $P_{\text{ND}} = P_{\text{ND}}(y(q), q)$. With probability $1 - q$ investors know that the manager has private information. Therefore in this case they set a price equal to $E_y$. If the investors have no information they set a price as explained in (5). The above formula is a convex-combination of both prices. Differentiating this condition with respect to $q$ yields:

\[ y' = \frac{\partial P_{\text{ND}}}{\partial q} + \frac{\partial P_{\text{ND}}}{\partial y} y' \]  

(15)

This is equivalent to

\[ y' = \frac{\frac{\partial P_{\text{ND}}}{\partial q}}{1 - \frac{\partial P_{\text{ND}}}{\partial y}} \]  

(16)

Therefore we have to calculate $\frac{\partial P_{\text{ND}}}{\partial q}$ and $\frac{\partial P_{\text{ND}}}{\partial y}$. First, we’ll derive $\frac{\partial P_{\text{ND}}}{\partial q}$. In order to do so, we start by computing the derivative with respect to $q$ of

\[ \frac{\partial}{\partial q} \left( \frac{(1 - p)qF_y}{p + (1 - p)qF_y} \right) = \frac{p(1 - p)F_y}{(p + (1 - p)qF_y)^2} \]  

(17)
and of
\[
\frac{\partial}{\partial q} \left( \frac{p}{p + (1-p)qF_y} \right) = \frac{-p(1-p)F_y}{(p + (1-p)qF_y)^2}. \tag{18}
\]

Using these derivatives we find that
\[
\frac{\partial P_{ND}}{\partial q} = (\mu - E_y) \frac{p^2}{(p + (1-p)qF_y)^2} > 0. \tag{19}
\]

Secondly, we derive \( \frac{\partial P_{ND}}{\partial y} \):
\[
\frac{\partial P_{ND}}{\partial y} = (1-q) \frac{\partial E_y}{\partial y} + q \left[ -\frac{(1-p)qf(y)}{(p + (1-p)qF_y)^2} \mu + AE_y + \frac{(1-p)qF_y}{p + (1-p)qF_y} \frac{\partial E_y}{\partial y} \right] \tag{20}
\]

where
\[
A = \frac{(1-p)qf(y)}{(p + (1-p)qF_y)^2}. \tag{21}
\]

Therefore
\[
\frac{\partial P_{ND}}{\partial y} = \left(1-q \right) \frac{(1-p)qF_y}{p + (1-p)qF_y} \frac{\partial E_y}{\partial y} + \left( E_y - \mu \right) \frac{(1-p)q^2f(y)}{(p + (1-p)qF_y)^2} < \frac{\partial E_y}{\partial y} \tag{22}
\]

For \( \frac{\partial E_y}{\partial y} < 1 \), which is the case for many distributions, we have \( \frac{\partial P_{ND}}{\partial y} < 1 \). Thus, it follows
\[
y' = \frac{\frac{\partial P_{ND}}{\partial q}}{1 - \frac{\partial P_{ND}}{\partial y}} > 0 \tag{23}
\]

**Discussion of the condition** \( \frac{\partial E_y}{\partial y} < 1 \)

Next, we want to show that the condition \( \frac{\partial E_y}{\partial y} < 1 \) holds for all distributions \( F(\cdot) \) with non-increasing density \( f(\cdot) \).
Lemma 1. Let \( F(\cdot) \) be a distribution with non-increasing density \( f(\cdot) > 0 \). Then we have \( \frac{\partial E_y}{\partial y} < 1 \).

Proof. Remember that
\[
E_y = \frac{\int_y^x f(x)dx}{F_y}.
\]

Therefore
\[
\frac{\partial}{\partial y} E_y = \frac{\partial}{\partial y} \frac{\int_y^x f(x)dx}{F_y} = \frac{yf(y)F_y - \int_x^y x f(x)dx \cdot f(y)}{F_y^2}.
\]

Since \( f(\cdot) \) is non-increasing we know that
\[
F_y = \int_x^y f(x)dx \geq \int_x^y f(y)dx = f(y)y - f(y)x.
\] (24)

Because
\[
\frac{\int_x^y x f(x)dx}{F_y} \geq x
\] (25)

we know that
\[
\frac{\partial}{\partial y} E_y = \frac{yf(y)F_y - \int_x^y x f(x)dx \cdot f(y)}{F_y^2} = \frac{yf(y)F_y}{F_y^2} - \frac{\int_x^y x f(x)dx}{F_y} \cdot \frac{f(y)}{F_y}
\]

Using (24) and (25) leads to
\[
\frac{\partial}{\partial y} E_y < \frac{(y - x)f(y)}{F_y} \leq 1.
\] (26)
Proof of Proposition 2. The nondisclosure set \( N^x \) as defined in (10) contains all private observations \( x \) that yield an expected nondisclosure price higher than \( x \). Thus, the equilibrium condition is

\[
y^x(q) = P(\hat{x} | x)
\]

(27)

where

\[
P(\hat{x} | x) = \frac{p}{p + (1 - p)qF_y} \mu + \frac{(1 - p)qF_y}{p + (1 - p)qF_y} E_y
\]

denotes the expected nondisclosure price if investors are oblivious of the manager’s information endowment. For readability and brevity of notation, in what follows we suppress the argument \( q \) and the superscript \( x \). Then differentiating the equilibrium condition (27) with respect to \( q \) yields:

\[
y' = \frac{\partial P(\hat{x} | x)}{\partial q} + \frac{\partial P(\hat{x} | x)}{\partial y} y'
\]

(28)

This is equivalent to

\[
y' = \frac{\frac{\partial P(\hat{x} | x)}{\partial q}}{1 - \frac{\partial P(\hat{x} | x)}{\partial y}}
\]

(29)

First, we derive \( \frac{\partial P(\hat{x} | x)}{\partial q} \). To do so, we start by computing the derivative with respect to \( q \) of

\[
\frac{\partial}{\partial q} \left( \frac{(1 - p)qF_y}{p + (1 - p)qF_y} \right) = \frac{p(1 - p)F_y}{(p + (1 - p)qF_y)^2}
\]

(30)
and of

\[
\frac{\partial}{\partial q} \left( \frac{p}{p + (1 - p)qF_y} \right) = \frac{-p(1 - p)F_y}{(p + (1 - p)qF_y)^2}. \tag{31}
\]

Using these derivatives we find that

\[
\frac{\partial P(\tilde{x}|\hat{x})}{\partial q} = (E_y - \mu) \frac{p^2}{(p + (1 - p)qF_y)^2} > 0. \tag{32}
\]

Interestingly, this is exactly \(-\frac{\partial P_{ND}}{\partial q}\) from the first setting. Secondly, we derive \(\frac{\partial P(\tilde{x}|\hat{x})}{\partial y}\):

\[
\frac{\partial P(\tilde{x}|\hat{x})}{\partial y} = -\frac{(1 - p)qpf(y)}{(p + (1 - p)qF_y)^2} + AE_y + \frac{(1 - p)qF_y}{p + (1 - p)qF_y} \frac{\partial E_y}{\partial y}. \tag{33}
\]

where

\[
A = \frac{(1 - p)qpf(y)}{(p + (1 - p)qF_y)^2}. \tag{34}
\]

Therefore

\[
\frac{\partial P(\tilde{x}|\hat{x})}{\partial y} = \left(1 - \frac{(1 - p)qF_y}{p + (1 - p)qF_y} \frac{\partial E_y}{\partial y} + (E_y - \mu) \frac{(1 - p)q^2pf(y)}{(p + (1 - p)qF_y)^2} \right. \left. < \frac{\partial E_y}{\partial y} \right) \tag{35}
\]

For \(\frac{\partial E_y}{\partial y} < 1\), which is the case for many distributions, we have \(\frac{\partial P(\tilde{x}|\hat{x})}{\partial y} < 1\). Thus, it follows that

\[
y' = \frac{\frac{\partial P(\tilde{x}|\hat{x})}{\partial q}}{1 - \frac{\partial P(\tilde{x}|\hat{x})}{\partial y}} < 0. \tag{36}
\]
References


