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Capitalized investments with entry and exit options and paradoxical tax effects

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Abstract Investment decisions are often characterized by uncertainty, irreversibility, and timing flexibility. We use a binomial model to investigate the interdependencies of effects from profit taxation and both an option to delay and an option to abandon on investment decisions. We show that increasing the tax rate can lead to paradoxical tax effects, i.e. it may foster an investor's willingness to invest. By contrast, if we abstract from the abandonment option, such paradoxical effects cannot be identified. Hence, we show that paradoxical tax effects can be caused by an abandonment option. Our results are helpful for investors facing risky investment opportunities and for improving typical valuation approaches.

Keywords Investment decisions · Real options · Tax effects · Timing flexibility · Uncertainty

JEL Classification H25 · H21

1 Introduction

In real world investment situations future cash flows are usually highly uncertain. Appropriate investment rules should hence account for that. If investors can react dynamically to possible states of nature, the degree of irreversibility and timing

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flexibility inherent in the projects in question should be integrated into the decision calculus. Moreover, it is well-known and has been a central issue in accounting and public finance research for many years that taxes can significantly affect investment decisions.

In recent years real option models have been widely accepted for assessing investment projects with stochastic cash flows (e.g., Dixit and Pindyck 1994; Trigeorgis 1996; Bertola 1998). These models have been extended with respect to taxes and allow us to develop after-tax decision rules for investment projects that are characterized by timing flexibility, uncertainty, and irreversibility. Thus, they enable us to account for the fact that investors cannot usually disinvest without costs once they realize a real investment project and then unexpectedly experience an unfavorable development of the investment environment. Furthermore, the investor may postpone the investment to a future point in time in the hope of better investment conditions, i.e. higher cash flows.

In this paper we investigate the interdependencies of effects from profit taxation on risky investment decisions and real options. We model an investment decision characterized by stochastic cash flows and an option to invest. Further, once the investment project is realized it includes an abandonment option. We show that increasing the tax rate can lead to paradoxical tax effects, i.e. may foster an investor's willingness to invest. By contrast, if we abstract from the possibility to abandon the investment object, we cannot identify such paradoxical effects.

To understand the mechanism of all involved effects and the economic intuition behind these effects, we determine the after-tax value of the option to enter the investment project with and without an abandonment option and finally receive an investment threshold or critical cash flow cut-off level. Evaluating the option to enter and simultaneously the option to abandon we derive the investor's after-tax decision rule. We find that the value of the option to abandon depends on the tax rate and on the periodical cash flows. That said, the tax effects are ambiguous. The option value can be an increasing or decreasing function in the tax rate. In contrast to classical tax paradoxa caused by tax timing effects as described in the literature, we find paradoxical patterns that are due to tax rate effects and the characteristics of the underlying investment object and that particularly depend on the existence of an inherent option to abandon.

This finding contributes to the stream of literature that explains potential sources of paradoxical tax effects under uncertainty. The resulting decision rules are helpful for investors facing risky investment opportunities. They help to forecast the impact of taxes on investment activities. Our results can be used to improve typical valuation approaches and hence are relevant to individual investors' tax planning. From the viewpoint of an investor, they can anticipate whether a risky project is discriminated, subsidized or treated neutrally by taxation. Hence tax planning is facilitated, i.e., it is easier for an investor to forecast the tax effects.

The remainder of the paper is organized as follows. After a brief literature review in Sect. 2 we introduce the reader to the basic features of the model in Sect. 3. In Sect. 4 we model the decision on the investment opportunity in the absence of the abandonment option as a benchmark situation and analyze the impact of taxation on the investment rule. For the benchmark scenario, we show that only normal, rather than paradoxical, effects occur. In Sect. 5 we expand the model framework with respect to an abandonment option at the second investment stage. We find that, unlike in the previous scenario, paradoxical tax effects can occur. We draw final conclusions in Sect. 6.

2 Literature

Several studies have analyzed whether and in what direction income and profit taxation distort individual and corporate investment decisions. The existence of so-called neutral tax systems that do not affect investment decisions have been proven under certainty and serve as a reference concept for analyzing tax effects. Prominent examples of such neutral tax systems are the cash flow tax and the taxation of true economic profit (e.g., Brown 1948; Samuelson 1964; Johansson 1969; Boadway and Bruce 1984 and Bond and Devereux 1995).

Integrating uncertainty, MacKie-Mason (1990) models nonlinear tax effects under uncertainty and demonstrates that policy may subsidize or discourage individual investment depending on the tax system. Alvarez et al. (1998) investigate whether or not tax policy uncertainty is harmful for investments in a dynamic stochastic adjustment model.[1] Altug et al. (2001) theoretically examine the implications of tax risk and persistence on irreversible investment decisions. Panteghini and Scarpa (2003) show that regulatory risk may or may not negatively affect investment decisions. Pawlina and Kort (2005) find that policy changes under uncertainty may have a non-monotonous impact on the investment threshold, whereas Bloom et al. (2007) point out that companies' responsiveness to any given policy is much lower in periods of high uncertainty.

This research highlights that more light should be shed on the interaction of investment decisions under uncertainty and tax effects and to derive elaborated investment rules that account for entry and exit options. Until now, the existing real option-oriented analyses that derive investment rules for risky investment projects with entry option and that account for tax effects have been rather limited (e.g., Agliardi 2001; Panteghini 2001, 2004, 2005; Niemann and Sureth 2004; Alvarez and Koskela 2008). Under specific assumptions in this context it has been possible to identify tax systems that are neutral with respect to investment decisions and may serve as a yardstick for measuring tax effects under uncertainty. For risk neutral investors, the existence of neutral tax systems has been proved in a real option context by Niemann (1999) and Sureth (2002). First results for neutral taxation under risk aversion were presented by Niemann and Sureth (2004). Moreover, there are a few analyses of tax effects in the real options framework that abstract from individual risk behavior, refer to risk neutral valuation and apply contingent claims analysis (e.g., Panteghini 2001; Niemann and Sureth 2002; Sureth 2002; Niemann and Sureth 2005; Sarkar and Goukasian 2006; Wong 2009). These studies focus on

[1] Problems created by anticipated tax reforms have been addressed by Alvarez et al. (1998) as well. These questions go back to King (1974) and later Auerbach and Hines (1988), Robson (1989), and Auerbach and Hassett (1992). In the following we abstract from such anticipatory and transitional problems.

investment projects that are traded on complete markets and hence fulfill the required spanning property. Using the real option framework, some investigations on the tax effects are restricted to numerical investigations (e.g., Pawlina and Kort 2005, p 1204).

Alvarez and Koskela (2008) focus on the impact of progressive taxation on irreversible investment and among other findings show that for sufficiently high volatilities, the investment threshold depends positively on volatility but negatively on the tax rate. The latter can be regarded as a tax paradox. Agliardi and Agliardi (2008, 2009) analyze the influence of different tax schemes on liquidation decisions. Furthermore, extending this contribution Wong (2009) shows that firms with an option to liquidate are encouraged to liquidate their operation earlier under progressive taxation as the corporate income tax rate rises. Thus, in the presence of tax progression and corporate income taxes holding decisions are distorted in a real option setting.

Beyond the well-known tax paradoxa under certainty caused either by depreciation allowances that exceed economic depreciation in present value terms or by loss carry forwards, minimum taxation or wealth taxation, Gries et al. (2007) pursue a general analytical approach to identify tax paradoxa under uncertainty in case of an option to invest. They point out that paradoxical tax effects can occur, i.e., a higher tax rate can lead to greater or in this specific context, earlier investments. In a scenario with an option to wait they show that the identified paradoxa are not due to tax scales or base effects but to uncertainty.

To date it has been Agliardi (2001) who analyzes the impact of a corporate cash flow tax and a subsidy to asset values on investments with entry and exit options and finds ambiguous effects on investment timing under this specific tax setting in a continuous time real option framework. Moreover, Niemann and Sureth (2009) investigate whether capital gains affect immediate and delayed investment asymmetrically under a combined exit-and-entry option for risky irreversible investment projects and uncertain cash flows. They finally show that taxing capital gains may induce a tax paradox. A more general analysis on tax effects from a profit tax for investments with entry and exit flexibility has not been performed yet. To fill the void we model a scenario in which the investor faces the opportunity to realize a non-depreciable investment project with stochastic cash flows. This project includes an option to delay the realization and also an option to abandon the risky project should the environment become unfavorable after realization. Then, we deduce investment rules for the given framework and analyze the possible tax effects on investment decisions.

3 Model

We consider an investor with an opportunity to invest in one of two mutually exclusive non-depreciable investment projects, one at time $t = 0$ and the other at $t = 1$. The investment object is a capitalized investment, e.g., an investment in property or in corporate stock with completely distributed earnings. The investment object neither increases nor decreases in value due to macroeconomic effects or speculative bubbles so that overall, no capital gains occur. As no capital gains have accrued in $t = 1$, capital gains taxation does not have to be considered.[2]

To optimize the decision the investor compares the after-tax costs and benefits from an immediate real investment with the expected costs and benefits of a delayed investment. The investor is assumed to be risk neutral and will carry out the project if a sufficiently high realization of the cash flow process at the time of decision can be observed. Alternatively, the investor will wait for better conditions and until then may invest funds in a capital market investment earning the risk-free market rate of return. Besides effects from uncertainty, taxation and more specifically the tax rate may asymmetrically affect an immediate real investment in comparison to a delayed risky real investment. This is all the more the case if the delayed investment offers the flexibility to react to future developments. More precisely, the value of a real option may be influenced by the tax rate in a non-linear fashion.

Unlike the Dixit-Pindyck type of real option model, e.g., Pawlina and Kort (2005), Gries et al. (2007) or Alvarez and Koskela (2008), uncertainty is modeled as the realization of a binary random variable in a one period model rather than a Brownian motion for an infinite time horizon. Thus, we are able to focus more on economic intuition. Again, an investor can choose between investing immediately ($t = 0$) or at some pre-specified future date ($t = 1$). While the cash flow from the investment can be observed at the time of decision ($t = 0$), future cash flows are subject to uncertainty. Hence, we have to refer to information about the time structure of future cash flows given by the binomial model to be able to decide between immediate or delayed investments.

The investor's pre-tax cost of capital is denoted by r. We assume that the tax system is characterized by a profit tax on income from real investment and a final tax on interest income.[3] Thus, profits from the real investment are subject to profit tax at tax rate τ. Losses at $t = 0$ or $t = 1$ can be completely offset at this tax rate τ, i.e., there is a tax refund in case of a negative tax base.[4] Interest payments are taxable or tax-deductible at a tax rate τ_f, i.e., $r_{\tau_f} = r(1 - \tau_f)$.[5]

[2] If the investor liquidates the project, they will receive the book value of the capitalized investment which is equal to the originally exogenously given initial outlay.

[3] Two separate tax rates rather than a uniform tax were assumed for mathematical convenience. To derive analytic (comparative static) results in the case of one uniform tax rate will be more complex because then the tax rate enters not only the numerator but also the denominator of the present value equation. Therefore, since the discounting factors increase in the tax rates models with one uniform tax rate for both profits from the investment and capital income, like interest income, are likely to lead to paradox settings as well. One even might expect that a uniform tax rate increases the size and frequency of paradoxical tax effects. To understand the mechanisms at work and the relevance of such settings it is important to investigate an expanded model with a uniform tax in detail in future research.

[4] This assumption of complete loss-offset can be justified by considering the investor to have positive cash flows from other sources that serve as loss compensation potential for the underlying project for tax purposes.

[5] Several countries levy a final tax on interest income. Austria has such a tax, and Germany introduced it at the beginning of 2009. Furthermore, the Nordic dual income tax systems are characterized by a preferential tax rate for all types of capital income. See, e.g., Nielsen and Sørensen (1997), Boadway (2004), Lindhe et al (2004), Sørensen (2005), and Kanniainen et al (2007).

Against this background, at $t = 0$ the risk neutral investor has two alternatives. Firstly, the investor can invest a fixed net amount I at $t = 0$.[6] Having realized the investment project at $t = 0$ the investor will receive the deterministic cash flow CF_0 at $t = 0$. Alternatively, the investor could decide at $t = 0$ on an investment to be realized at $t = 1$. Investing later requires an effective net cash outlay of βI at $t = 1$, where β is some exogenously given growth parameter.[7] However, the decision on the delayed project has to be made at $t = 0$, so the project must be initiated at the same time as the immediate project.

The profit tax system in our model is similar to a sales tax as the tax base is simply the cash flow, i.e. the net of positive and negative cash flows from turnover and expenses. Since we do not specify in detail how the cash flows are composed our model in this respect is suitable for both a profit and a sales tax. Under a profit tax, in case of an investment object that is non-depreciable the initial investment outlay affects profit taxation only if the acquired asset or stock is sold. Then it can be deducted from the price the seller receives and the remaining capital gain may be subject to a capital gains tax. In our model we explicitly abstract from a capital gain and hence capital gains tax. We do not abstract from the implications of the initial outlay. Rather than explicitly modeling this issue we implicitly take account of the initial outlay.[8] Keeping these assumptions in mind, the after-tax cash flows in the model also implicitly account for the effects from the initial outlay realized if the investment is sold.

An example of such an investment is given by R&D investments by investors or venture capitalists who have decided to conduct a developmental project (e.g. developing an optical storage device). The $t = 0$ investment could be a Blu-ray device while the $t = 1$ investment would be some more advanced device with higher storage capacity. Postponing the investment project even further to a future time with $t > 1$ is not possible since the venture capitalists would only commit themselves to this development project if their capital is invested into the real investment project at $t = 0$ or $t = 1$. At a later point of time this investment opportunity is no longer available.

We assume that the investor evaluates both alternative investments (immediate and delayed investment) based on their expected after-tax net present value (NPV). An investment at $t = 0$ in our one-period model leads to a deterministic cash flow of CF_0 with $CF_0 > 0$, while an investment at $t = 1$ results in a stochastic cash flow $\widetilde{CF}_1$. In case of the good state of nature G the cash flow from the delayed project equals $\overline{CF_1} = \alpha(CF_0 + 1)$, while it is $\underline{CF_1} = \alpha(CF_0 - 1)$ in case of the bad state of

[6] We assume an initial investment of $\hat{I}$ at $t = 0$ and that the investor liquidates the project in the subsequent period and hence receives the book value of the capitalized investment $\hat{I}$ at $t = 1$. Discounting the book value and deducting this present value of the book value from the initial investment $\hat{I}$ leads to the initial effective net investment outlay I with $I := \hat{I} - \frac{\hat{I}}{1+r_{\tau_f}} = \hat{I}\left(1 - \frac{1}{1+r_{\tau_f}}\right)$. For simplicity we focus in the following on investing the initial effective net investment outlay I.

[7] In line with an immediate investment for the delayed investment we implicitly assume that $\beta I := \beta\hat{I} - \frac{\beta\hat{I}}{1+r_{\tau_f}} = \beta\hat{I}\left(1 - \frac{1}{1+r_{\tau_f}}\right)$.

[8] See footnote 6 and 7.

nature B. Both states of nature are equally likely, i.e., their probability is $p = \frac{1}{2}$. Therefore, the expected value of the pre-tax cash flow of an investment in period $t = 1$ is $E[\widetilde{CF_1}] = \alpha CF_0$. Consequently, the parameter α, with $\alpha > 0$, can be interpreted as a growth factor of the (expected) cash flows between period 0 and period 1. In order to keep the model transparent and to avoid unnecessary case distinctions we assume that $\alpha < 1 + r_{\tau_f}$.[9] That is, the cash flow growth rate is below the investor's cost of capital.

The investor cannot anticipate the state of nature at $t = 1$ in $t = 0$, i.e., when the choice between immediate and delayed investment is made. Thus the investor faces the following investment strategies:

(1) invest immediately and receive the deterministic cash flow at $t = 0$ (invest now), or
(2) invest later and receive the stochastic cash flow at $t = 1$ (invest later without exit flexibility). The investor decides to delay the investment and invest in $t = 1$. We abstract from the possibility to abandon the investment. Thus, the investor cannot react on the extra information available at $t = 1$. Hence, a potential investment decision at $t = 0$ for an investment at $t = 1$ is irreversible (benchmark scenario for a delayed investment);
(3) invest later and exercise the option to abandon (invest later with exit flexibility to abstain from delayed investment). The investor decides to delay the investment to $t = 1$. In contrast to (2), we include an abandonment option at $t = 1$ for the $t = 1$ investment project. Abandoning will eliminate the cash flow in $t = 1$. The salvage value equals the necessary investment outlay and therefore formally no net investment occurs if the exit option is exercised. More concretely, if the exit option is not exercised the gains from a $t = 1$ investment equal $\alpha(CF_0 + 1) - \beta I \geq 0$ in the good state of nature and $\alpha(CF_0 - 1) - \beta I \leq 0$ in the bad state. If the option is exercised, the gains are zero.

We abstract from an option to abandon at the first stage of the analysis and regard the outlined scenario with an entry option only (investment strategies (1) and (2)) as a benchmark scenario for analyzing later the effects of an exit option. Then, at the second stage of our investigation we model a scenario that comprises an abandonment option (investment strategies (1) and (3)).

Against this background we analyze how taxes influence investor behavior (investment, divestment). Do taxes foster an investor's willingness to remain invested? Do taxes hinder real investment? Do taxes influence the timing and duration of an investment and in turn, the timing of divestment?

To identify how taxes affect investment behavior we have to distinguish between normal, non-distorting, and paradoxical effects. If taxes are not neutral with respect to investment decisions but distortive, typically we expect that levying taxes on profits from real investment will decrease an investor's willingness to invest

[9] In the following we focus on scenarios with $\alpha < 1 + r_{\tau_f}$ to keep the model simple. For reasons of completeness and to show that this does not restrict the generality of our results we have inserted a consideration for the case $\alpha > 1 + r_{\tau_f}$ in Appendix 2. In case of $\alpha = 1 + r_{\tau_f}$ the size of CF_0 never influences the optimal investment choice. Therefore, only normal tax effects occur.

(normal effect). By contrast, under a neutral tax system taxation would not affect investment behavior at all. Further, if investors are more willing to realize real investment projects that are subject to tax than a tax-free alternative, the tax effect is referred to as paradoxical. Such paradoxical effects are well-known under certainty and are caused either by depreciation allowances that exceed economic depreciation in present value terms or investment credits[10] or by loss carry forwards, minimum taxation or wealth taxation.[11]

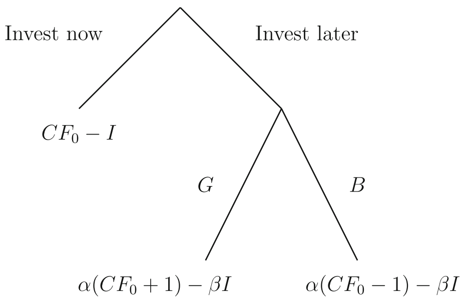
In the following section we investigate the impact of taxes on the investment decision. We will see that paradoxical tax effects do not occur in the benchmark case while they may arise if an abandonment option is available. Therefore, we will be able to conclude that paradoxical tax effects can emerge in the presence of real options, particularly if the investment includes an abandonment option.

4 No flexibility to abandon the investment

To analyze the impact of taxation on the investment decision in $t = 0$ we focus on an option to wait only (investment strategies (1) and (2)) as a benchmark case for further investigations. We assume that the option has a strictly positive value and therefore affects the decision calculus.

The sequence of events and the decision problem in our benchmark scenario without an abandonment option are illustrated in Fig. 1.

Fig. 1 Decision tree in the benchmark case (no option to abandon)



At $t = 0$ the investor can either invest immediately or delay the investment until $t = 1$, and until then invest in the capital market. By assumption, deciding to postpone the investment at $t = 0$ the investor simultaneously accepts the obligation to realize the project at $t = 1$. Consequently, at $t = 1$ there is no longer a default alternative if the investor has refrained from immediate investment at $t = 0$ and has committed to postponing the investment until $t = 1$. Having decided to delay the investment the investor cannot react to new information at $t = 1$. Figure 1 shows that the investor can avoid losses exceeding $NPV = CF_0 - I$ with $CF_0 > 0$. Since the cash flow CF_0 is assumed to be positive the investor can limit their loss to $NPV > -I$ if they invest immediately.

[10] See Samuelson (1964), MacKie-Mason (1990).

[11] See, e.g., Auerbach and Poterba (1987), p 319, 336; Sureth and Maiterth (2008).

Abstracting from an option to abandon at the first stage of the analysis we identify settings in which only normal tax effects occur.

An immediate investment of I at $t = 0$ yields a cash flow of CF_0 at date $t = 0$, and the surplus from the investment is subject to a profit tax at tax rate τ. The investment yields after-tax profits (or losses) P_0 with

$$P_0 = (1 - \tau)CF_0 - I. \tag{1}$$

Alternatively, the investment can be delayed to $t = 1$ but then must definitely be carried out. At $t = 1$ two equally probable states are possible. Investing βI leads to either $\overline{CF_1}$ in the good state or $\underline{CF_1}$ in the bad state.[12] Since there is no possibility to abandon the investment at $t = 1$, the expected profit in present value terms of a delayed project is

$$E\left[\frac{\widetilde{P_1}}{1 + r_{\tau_f}}\right] = (1 - \tau)\frac{\alpha}{1 + r_{\tau_f}}CF_0 - \frac{\beta I}{1 + r_{\tau_f}}. \tag{2}$$

Using Eqs. (1) and (2), we find the investment is delayed if and only if the expected return from delayed investment in present value terms is greater than the return from an immediate investment. Consequently, using our assumption $\alpha < 1 + r_{\tau_f}$, delayed investment is optimal if and only if

$$CF_0 \leq \frac{I\left(1 - \frac{\beta}{1 + r_{\tau_f}}\right)}{(1 - \tau)\left(1 - \frac{\alpha}{1 + r_{\tau_f}}\right)}. \tag{3}$$

We denote the corresponding threshold or cut-off level by CF_0^*. That is, since we have to take into account that the cash flows CF_0 are positive,

$$CF_0^* = \max\left\{0, \frac{I\left(1 - \frac{\beta}{1 + r_{\tau_f}}\right)}{(1 - \tau)\left(1 - \frac{\alpha}{1 + r_{\tau_f}}\right)}\right\}. \tag{4}$$

This result can be interpreted as follows. Since the cash flow grows at a lower rate than the firm's cost of capital (i.e. $\alpha < 1 + r_{\tau_f}$), it follows that higher cash flows favor early investments. An immediate investment is chosen for all positive values of CF_0 with $CF_0 \geq CF_0^*$. For lower values of CF_0 the investment is postponed to $t = 1$. Since delayed investments can be interpreted as a decrease in the investor's willingness to invest, we have normal tax effects if CF_0^* increases in τ. Contrary, if CF_0^* decreases in τ we will have fewer delayed and more immediate investments and consequently paradoxical tax effects caused by the underlying profit tax.

If the growth rate β of the investment outlay is below the firm's cost of capital ($\beta < 1 + r_{\tau_f}$), it follows from Eq. (4) that

[12] Since we have assumed an interest rate of r_{τ_f}, an immediate investment of I corresponds to an investment of $(1 + r_{\tau_f})I$ at $t = 1$. However, in this section we make no assumption about the relation of the growth factor β to $1 + r_{\tau_f}$. By contrast, we will assume $\beta > 2(1 + r_{\tau_f})$ in the following section in order to simplify the investigation and focus on first-order effects.

$$CF_0^* = \frac{I\left(1 - \frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1 - \frac{\alpha}{1+r_{\tau_f}}\right)}. \tag{5}$$

In this case we have a strictly positive value of the cut-off level. Again, this can be explained by realizing that $\alpha < 1 + r_{\tau_f}$. Since the discounted value of outlay for a delayed investment is smaller than the required initial outlay for an immediate investment, postponing the investment is attractive at least for small values of CF_0. It is obvious that in this case the critical cash flow threshold CF_0^* increases in τ and therefore we have normal tax effects.

If $\beta > 1 + r_{\tau_f}$, it follows that the second term under the max-operator in Eq. (4) is negative. Therefore, we have $CF_0^* \equiv 0$ for all τ and hence no distorting effects from the underlying profit tax. Note that here, neutrality is due to the assumption of positive cash flows.[13]

Proposition 1 *The optimal investment strategy in the setting described above is as follows:*

1. *If $\beta < 1 + r_{\tau_f}$, the investor prefers to delay the investment for all $CF_0 \in [0, CF_0^*)$, where $CF_0^* > 0$. They are indifferent for $CF_0 = CF_0^*$ and prefer early investment* for $CF_0 > CF_0^*$.
2. *If $\beta = 1 + r_{\tau_f}$, the investor can choose to either invest or delay the investment for $CF_0 = CF_0^* \equiv 0$, but prefers early investment for $CF_0 > CF_0^* \equiv 0$.*
3. *If $\beta > 1 + r_{\tau_f}$, the investor never delays the investment. This corresponds to $CF_0 = CF_0^* \equiv 0$.*

Therefore, since the function

$$CF_0^* = \max\left\{0, \frac{I\left(1 - \frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1 - \frac{\alpha}{1+r_{\tau_f}}\right)}\right\} \tag{6}$$

does not decrease in τ, paradoxical tax effects never can occur for this benchmark investment problem.

More specifically,

1. *normal tax effects occur for $\beta < 1 + r_{\tau_f}$ and*
2. *no distorting tax effects occur for $\beta \geq 1 + r_{\tau_f}$.*

One of our crucial assumptions is a tax system with a profit tax. We show in Appendix 1 that a cash flow tax implies neutral tax effects in our setting irrespective of assumption on β.

[13] If negative cash flows were possible we would have $CF_0^* = \frac{I\left(1 - \frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1 - \frac{\alpha}{1+r_{\tau_f}}\right)}$ and therefore CF^* would be decreasing in τ for $\beta > 1 + r_{\tau_f}$. This in turn implies paradoxical tax effects. Note that we assume that one of the two investments has to be realized and therefore negative cash flows CF_0 do not imply no investment. To conclude, under the given set of assumptions paradoxical tax effects can generally occur. However, in order to focus on the effect of the abandonment option we exclude cases with negative cash flows by assuming that $CF_0 > 0$.

Up to now our analysis has focussed on the case $\alpha < 1 + r_{\tau_f}$. The complementary case ($\alpha > 1 + r_{\tau_f}$) is quite similar. Since most effects are similar, we will not refer to this case in the investigation on the abandonment option but have included details about this case in Appendix 2.

To summarize, we find that in our benchmark investment scenario where the investment does not include an option to abandon, in general no paradoxical tax effects arise. In the following section we expand our model framework to include an option to abandon a delayed investment after the investor has observed the state of nature and show that paradoxical tax effects can occur.

5 Flexibility to abandon the investment

Integrating an option to abandon, we prove that there are situations that lead to paradoxical tax effects. Our analysis clarifies that these paradoxical tax effects are caused by the presence of the underlying (abandonment) real option.

The events and the decision tree in case of the extended scenario with an abandonment option are illustrated in Fig. 2.

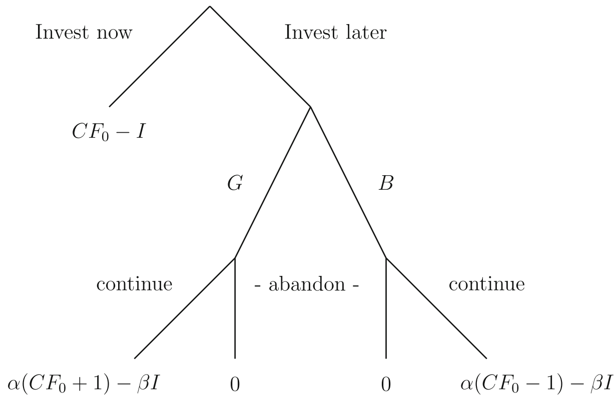


Fig. 2 Decision tree in the presence of the abandonment option

The events are fairly similar to the benchmark case presented before. The investor can choose to invest at $t = 0$ (invest now) or schedule an investment for $t = 1$ (invest later). In case of an investment at the later date, the state of nature can be observed at $t = 1$. In contrast to the previous scenario, the investor can now abstain from the originally planned delayed investment and exercise the option to abandon it. In case of an abandonment, on the one hand the investor does not receive the cash flows from the real investment project, but on the other faces no initial outlay βI and in turn, realizes neither gains nor losses.[14] If the investor holds the exit option and thus carries out and keeps the investment project, they have to invest an amount of βI and realize cash flows $\widetilde{CF_1}$ as in the benchmark case.

[14] This assumption follows from mathematical convenience. In principle, a capital gain or loss could occur from abandonment. See Sect. 3, footnote 2 as well.

First, we assume that $\alpha(CF_0 + 1) - \beta I \geq 0$. This ensures that an investment at $t = 1$ is not abandoned in the good state of nature for at least low tax rates τ. We analyze the investment problem by backward induction and consider tax rates τ for which the $t = 1$ investment is not abandoned in the good state. If the tax rate τ is sufficiently low, the investment will not be abandoned in the good state of nature at $t = 1$. For all other values of the tax rate, it will.

Second, since it is possible to abandon the investment (with a salvage value that equals the necessary investment), the investor terminates the project in the bad state. Here, the assumption $\alpha(CF_0 - 1) - \beta I \leq 0$ is crucial.

The value of the $t = 1$ investment equals the expected present value of the investment in the two states. Therefore, taking into account the optimal execution of the abandonment option the present value of a postponed investment is given by

$$\frac{1}{2}\left[(1-\tau)\alpha\frac{CF_0+1}{1+r_{\tau_f}} - \beta\frac{I}{1+r_{\tau_f}}\right]. \tag{7}$$

Hence, a necessary criterion for delaying the project is that

$$\frac{1}{2}\left[(1-\tau)\alpha\frac{CF_0+1}{1+r_{\tau_f}} - \beta\frac{I}{1+r_{\tau_f}}\right] \geq (1-\tau)CF_0 - I. \tag{8}$$

This condition can be rewritten as

$$I\left(1-\frac{1}{2}\frac{\beta}{1+r_{\tau_f}}\right)+\frac{1}{2}\frac{1-\tau}{1+r_{\tau_f}}\alpha \geq (1-\tau)CF_0\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right). \tag{9}$$

Remember the assumption $\alpha < 1 + r_{\tau_f}$. Since this assumption implies $\alpha < 2(1 + r_{\tau_f})$, condition (9) is equivalent to

$$CF_0 \leq \frac{I\left(1-\frac{1}{2}\frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)}+\frac{1}{2}\frac{\frac{\alpha}{1+r_{\tau_f}}}{\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)}. \tag{10}$$

We denote the corresponding threshold or cut-off level by CF_0^*. Since we have to take into account that the cash flows CF_0 are positive, that is

$$CF_0^* = \max\left\{0, \frac{I\left(1-\frac{1}{2}\frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)}+\frac{1}{2}\frac{\frac{\alpha}{1+r_{\tau_f}}}{\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)}\right\}. \tag{11}$$

Figure 3 depicts the cut-off level by CF_0^* as a function of τ for $\alpha = 1$, $\beta = 2.1$, $r = 0.03$ and three different initial outlays $I = 9$, 10 and 11.

The optimal investment decision and especially the cut-off level CF_0^* can be explained as follows. In line with the benchmark case the cash flow grows at a lower rate than the firm's cost of capital (i.e. $\alpha < 1 + r_{\tau_f}$). Therefore, it is obvious from Eq. (8) that higher cash flows favor early investments. A $t = 0$ investment is chosen for all values of CF_0 that are higher than CF_0^*. For values of CF_0 with $CF_0 \geq 0$ and $CF_0 < CF_0^*$ the investment is postponed to $t = 1$. Again, delayed investments can be interpreted as a decrease in the investor's willingness to invest. Therefore, we have normal tax effects if CF_0^* increases in τ. By contrast, if CF_0^* decreases in τ we would

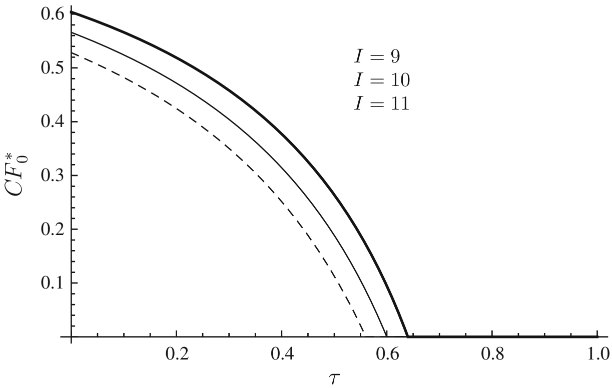


Fig. 3 The cut-off level CF^* as a function of the tax rate τ

have fewer delayed and instead more immediate investments for rising tax rates and consequently paradoxical tax effects.

Unlike in the benchmark case without flexibility to abandon, the equation for strictly positive values of CF_0^* consists of two parts.[15] In a sense the first part

$$\frac{I\left(1 - \frac{1}{2}\frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1 - \frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)} \tag{12}$$

is similar to the equation of the cut-off level CF_0^* in the benchmark case that was given by[16]

$$\frac{I\left(1 - \frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1 - \frac{\alpha}{1+r_{\tau_f}}\right)}.$$

The fractions $\frac{1}{2}$ in the numerator and denominator of Eq. (12) have to be inserted because a $t = 1$ investment will be abandoned in the bad state and therefore the investment is only conducted with a probability of $\frac{1}{2}$. This is also reflected by the left hand side of inequality (8) or by inequality (9).[17]

The second term of the cut-off level CF_0^*

$$\frac{1}{2}\frac{\frac{\alpha}{1+r_{\tau_f}}}{\left(1 - \frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)}$$

reflects that at $t = 1$ the cash flows from real investments are higher in the good state than at $t = 0$ (i.e., $CF_0 + 1$ instead of CF_0). This "gain" is part of the cash flow and is taxed at the same rate as the whole cash flow. Consequently, all tax

[15] If CF_0^* is strictly positive it equals $\frac{I\left(1-\frac{1}{2}\frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)} + \frac{1}{2}\frac{\frac{\alpha}{1+r_{\tau_f}}}{\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)}$.

[16] See Eq. (5).

[17] If the investment project was depreciable the effects emerging from this term would be smaller.

components vanish with respect to this term and thus the second term is independent of the tax rate τ.[18] This independency contrasts with the first term that non-trivially depends on the tax rate τ. Here, the dependency on the tax rate τ is due to all investments being capitalized and non-depreciable and therefore having no influence on the periodical tax base.[19]

Further, note that

$$\frac{1}{2}\frac{\frac{\alpha}{1+r_{\tau_f}}}{\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)}>0 \tag{13}$$

due to our assumption concerning α. In addition, if $\beta > 2(1+r_{\tau_f})$ is satisfied,

$$\frac{I\left(1-\frac{1}{2}\frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1-\frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)} \tag{14}$$

decreases in τ which leads to *paradoxical tax effects*.

Finally, if the $t = 1$ investment is also abandoned in the good state of nature, which can occur if the tax rate is sufficiently high, the present value of this investment taking into account the optimal execution of the abandonment option is given by zero. Therefore, a necessary criterion for delaying the project is that $0 \geq (1-\tau)CF_0 - I$ or

$$CF_0 \leq \frac{I}{1-\tau}. \tag{15}$$

In this case we have normal tax effects.

In the following proposition we summarize these results.

Proposition 2 *If* $\alpha < 1 + r_{\tau_f}$ *and* $\beta > 2(1+r_{\tau_f})$, *then we have paradoxical tax effects in the presence of the option to abandon.*

To interpret the above proposition it is firstly helpful to provide some economic intuition for this setting and secondly to focus on the effects of the option to abandon.

First, intuitively a setting with $\alpha < 1 + r_{\tau_f}$ and $\beta > 2(1+r_{\tau_f})$ is likely for all export-oriented industries, at least in the short run. For instance, it is given for the German automotive industry which sells its products in the US. If the US dollar weakens against the euro and if the products are manufactured in Germany and thus input prices are driven by local cost, β will exceed α. In this case US revenues may undergo only a slight increase or even decrease while production costs may rise in Germany. A similar argument is valid for oil-producing countries in the Middle East. Their costs are mainly based on the euro, because these countries mainly hire European companies while revenues are denominated in US dollars. We refer to settings with the expected growth rate of investment expenditures being a multiple

[18] To see this in detail concentrate on the term "+1" at the left side of inequality (8) and follow this term through inequalities (8) to (10).

[19] Again, follow the corresponding terms through inequalities (8) to (10). Further, see footnote 5.

of the expected growth rate of the cash flows. Since the investment project's only benefit is the expected cash flow, often both growth rates will be closely related. In the long run they can even be presumed to be identical. Nevertheless, in the short run, particularly for R&D companies, such settings are likely to occur.[20]

Second, in our model we define the value of the option to abandon as the value of the flexibility associated with the possibility to abandon. The expected net present value from a delayed investment in the absence of the abandonment option V^{abs} is

$$V^{abs} = (1-\tau)\frac{\alpha}{1+r_{\tau_f}}CF_0 - \frac{\beta I}{1+r_{\tau_f}}, \tag{16}$$

while the value of a delayed investment in the presence of the abandonment option V^{pres} is

$$V^{pres} = \frac{1}{2}\left[(1-\tau)\frac{\alpha(CF_0+1)}{1+r_{\tau_f}} - \frac{\beta I}{1+r_{\tau_f}}\right]. \tag{17}$$

The value of the flexibility V^{op} to abandon can be described by

$$\begin{aligned} V^{op} &= V^{pres} - V^{abs} \\ &= (1-\tau)\left(\frac{1}{2}\frac{\alpha}{(1+r_{\tau_f})}[1-CF_0]\right) + \frac{1}{2}\frac{\beta I}{1+r_{\tau_f}}. \end{aligned} \tag{18}$$

It must be considered that the above equation is only valid under the assumption that the abandonment option is exercised in the bad state and not in the good state.[21] For parameters for which this execution pattern is optimal the above value difference V^{op} will be positive and therefore the option will always have a positive value. Obviously, the value of the option to abandon decreases in τ as long as $CF_0 < 1$ and increases as long as $CF_0 > 1$.

Whereas profit taxation only affects the cash flow component in the net present value equation of both alternative investments, with respect to the value of flexibility exercising the option to abandon affects both the cash flows and the term including the investment outlay. Specifically, the option is exercised whenever cash flows do not justify the investment costs. As positive cash flows still can be too low to compensate for the initial outlay in present value terms the exit option will be exercised. Therefore, the expected value of the project may increase in the presence of the exit option. The first term in Eq. (18) captures the effect of the expected cash flow. The second term reflects the effect from the investment outlay.

If $CF_0 < 1$ a negative cash flow can be avoided by abandoning the investment. Since the tax system provides a complete loss offset for a negative tax base (losses) and exercising the abandonment option eliminates the possible benefit from a loss-

[20] Furthermore, speculative bubbles like those during the dot.com boom in the late 1990s can be considered as another example for divergent growth rates and thus represent another example for settings that may lead to tax paradoxa.

[21] This assumption restricts possible values of CF_0 to an interval. The investment is abandoned in the bad state if CF_0 is sufficiently small and it is not abandoned in the good state if CF_0 is sufficiently high. Therefore, CF_0 must be between certain bounds that are functions of the other parameters of the model. However, this assumption does not limit the generality of our model, since only extreme cases are excluded.

induced tax refund, this positive effect decreases with the tax rate. Therefore the value of the option decreases with the tax rate. This mechanism is the reason for the paradoxical tax effects. Hence, the occurrence of such effects is due to the assumption that the tax system provides a complete loss offset. However, in line with our definition of paradoxical effects, a rise in the tax rate makes the immediate investment including the option to abandon more attractive.

By contrast, for $CF_0 > 1$ a positive cash flow is eliminated if the exit option is exercised. This negative effect decreases with the tax rate. Hence, the value of the option increases in the tax rate τ, because eliminating the positive cash flow is less harmful for the investor if this cash flow had been subject to high tax rates.

For further intuition, let NPV_0 denote the net present value of an immediate investment and let NPV_1^{abs} denote the expected net present value of a delayed investment in the absence of the abandonment option. Specifically, we have[22]

$$NPV_0 = (1-\tau)CF_0 - I \quad \text{and}$$
$$NPV_1^{abs} = (1-\tau)\frac{\alpha}{1+r_{\tau_f}}CF_0 - \frac{\beta I}{1+r_{\tau_f}}. \tag{19}$$

Correspondingly, by NPV_1^{pres} we denote the net present value of a delayed investment with an abandonment option[23]

$$NPV_1^{pres} = \frac{1}{2}\left[(1-\tau)\frac{\alpha(CF_0+1)}{1+r_{\tau_f}} - \frac{\beta I}{1+r_{\tau_f}}\right]. \tag{20}$$

The difference $DIFF^{abs}$ between a delayed and an early investment in the absence of the real option is given by

$$\begin{aligned} DIFF^{abs} &= NPV_1^{abs} - NPV_0 \\ &= (1-\tau)CF_0\left[\frac{\alpha}{1+r_{\tau_f}} - 1\right] - I\left[\frac{\beta}{1+r_{\tau_f}} - 1\right]. \end{aligned} \tag{21}$$

The investment will be delayed whenever $DIFF^{abs}$ is positive. According to our assumptions about α and β in this section, $\alpha < 1 + r_{\tau_f}$ and $\beta > 2(1 + r_{\tau_f})$, the difference will always be negative (see also proposition 1 (3)).

Furthermore, the difference $DIFF^{pres}$ between delayed and early investment in case of an option to abandon at $t = 1$ is

$$\begin{aligned} DIFF^{pres} &= NPV_1^{pres} - NPV_0 \\ &= (1-\tau)CF_0\left[\frac{1}{2}\frac{\alpha}{1+r_\tau} - 1\right] + (1-\tau_f)\frac{1}{2}\frac{\alpha}{1+r_f} - I\left[\frac{1}{2}\frac{\beta}{1+r_{\tau_f}} - 1\right] \\ &= DIFF^{abs} + V^{op}. \end{aligned} \tag{22}$$

Obviously, $DIFF^{pres}$ decreases in the cash flow CF_0. As $DIFF^{abs} < 0$, it can only be positive if the value of the option is sufficiently large. Since the value of the real

[22] Note that NPV_1^{abs} is another notation for V^{abs}. See Eq. (16).

[23] Note that NPV_1^{pres} is another notation for V^{pres}. See Eq. (17).

option decreases for $(CF_0 < 1)$,[24] the difference also decreases. Therefore, higher tax rates induce more early investments.

6 Conclusions

Our investigation focuses on the influence of tax rates on investment decisions under uncertainty and timing flexibility. In this paper we study investment decisions concerning a real investment at two different points in time. We assume that the underlying investment has to be capitalized. As it is non-depreciable by assumption it does not imply a reduction in the tax base thanks to depreciation allowances. If we find that the investor prefers to delay the investment, we interpret this as a low willingness to invest (immediately). Analyzing the influence of taxes on investor behavior, we look for scenarios with taxes that foster investment activities, leave investment activities unaffected or discriminate investment activities. In our model a tax effect is considered normal if higher tax rates induce a postponement. If an increase in the tax rate does not influence investment timing, we refer to it as a non-distorting tax. By contrast, if higher tax rates lead to earlier investments, we have paradoxical tax effects.

Assuming the investor faces two options, an option to wait and an option to abandon, we regard a scenario without an option to abandon as the benchmark case. Here, it turns out that only non-distorting or normal tax effects on investment timing and thus an investor's willingness to invest occur. Finally, we receive an investment threshold or critical cash flow cut-off level for a scenario with an abandonment option. Evaluating the option to enter and simultaneously the option to abandon, we derive the investor's after-tax decision rule. We find that the value of the option to abandon depends on the tax rate and on the periodical cash flows. The option value can be an increasing or decreasing function in the tax rate. Hence, in the presence of the abandonment option, we find scenarios with paradoxical tax effects. We show that the observed paradoxical tax effects are due to the presence of the real abandonment option itself.

This finding contributes to the stream of literature that explains potential sources of paradoxical tax effects. Our result is due to the fact that the value of the real abandonment option depends on the tax rate. More precisely, if the cash flows are small, the value of the option decreases with a rise in the tax rate. This is because when exercising the option to abandon and cash flows are small, abstaining from the real investment eliminates negative cash flows that would have been realized otherwise. As negative cash flows reduce the tax base or even lead to a negative tax base and hence a tax refund, the value of the option to abandon decreases in the tax rate. Consequently, higher tax rates induce earlier investment and therefore a boost in the investor's willingness to invest.

The resulting decision rules are helpful for investors facing risky investment opportunities. They help to forecast the impact of taxes on investment activities.

[24] It can be shown, using our assumptions of this section about α and β, that scenarios with $CF_0 < 1$ are the decisive outcomes for the cash flow at time $t = 0$ for our investigation.

Our results are relevant to individual investors' tax planning. Furthermore, we highlight the overwhelming importance of integrating taxes in typical valuation approaches.

For future research on tax effects under uncertainty, our model can be extended with respect to more complex tax rules. For instance, asymmetric taxation of ordinary income and capital gains could be integrated into this approach by inserting exogenous or, in case of depreciable investment objects, even endogenous liquidation proceeds. Asymmetric taxation of gains and losses could be integrated by introducing a separate (lower) tax rate for losses representing loss offset restrictions, yielding testable hypotheses for empirical or quasi-experimental investigations.

Appendix 1: Cash flow tax implies tax neutrality

We show in the following that the cash flow tax which is known to be neutral under certainty implies tax neutrality in our setting as well. If we had a cash flow tax[25] instead of a profit tax, the cut-off level CF_0^* would not depend on τ and hence there would be no interdependence between the taxation and the investment problem under uncertainty and timing flexibility emerges. This can be seen by noting that the delayed investment is preferred if and only if

$$(1-\tau)\left[\frac{\alpha}{1+r_{\tau_f}}CF_0 - \frac{\beta I}{1+r_{\tau_f}}\right] \geq (1-\tau)[CF_0 - I]. \tag{23}$$

Note that the term $(1-\tau)$ vanishes. Therefore, it can be argued as above that a similar cut-off level CF_0^* as in the main body of the text satisfies

$$CF_0^* = \max\left\{0, \frac{I\left(1-\frac{\beta}{1+r_{\tau_f}}\right)}{\left(1-\frac{\alpha}{1+r_{\tau_f}}\right)}\right\}. \tag{24}$$

It is obvious from the above equation that under a cash flow tax the cut-off level does not depend on the tax rate τ.

Appendix 2: The case $\alpha > 1 + r_{\tau_f}$ in the benchmark scenario

Furthermore, we could include an analysis of the case $\alpha > 1 + r_{\tau_f}$ at this point. For the sake of completeness, we briefly sketch the arguments for such a setting here. For $\alpha > 1 + r_{\tau_f}$ we receive from Eqs. (1) and (2) corresponding to Eq. (3) that

$$CF_0 \geq \frac{I\left(1-\frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1-\frac{\alpha}{1+r_{\tau_f}}\right)}. \tag{25}$$

[25] Note that a cash flow tax has been proven neutral for risk neutral investors in a real option setting, cf. Niemann and Sureth (2004).

Again, we denote the corresponding cut-off level by CF_0^*. That is for positive cash flows

$$CF_0^* = \max\left\{0, \frac{I\left(1 - \frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1 - \frac{\alpha}{1+r_{\tau_f}}\right)}\right\}. \tag{26}$$

The two equations above can be interpreted as follows. Contrary to the case $\alpha < 1 + r_{\tau_f}$, the cash flow grows at a higher rate than the firm's cost of capital (i.e. $\alpha > 1 + r_{\tau_f}$). Therefore, it is obvious from Eqs. (1) and (2) that higher cash flows favor delayed investments. A postponed investment will be chosen for all values of CF_0 in the interval $[CF_0^*, \infty)$. For lower values of CF_0^* more possible investments are delayed to $t = 1$. Since we associate delayed investments with fewer investments, we find normal tax effects if CF_0^* decreases in τ. Contrary, if CF_0^* increases in τ, we get fewer delayed and therefore more early investments and consequently paradoxical tax effects. Finally, it can be argued as above that for $\beta > 1 + r_{\tau_f}$ normal tax effects occur and that in all other cases ($\beta \leq 1 + r_{\tau_f}$) the tax is neutral.

Appendix 3: Paradoxical tax effects in case of state-dependent abandonment values

In the following, we investigate the same investment setting, except that the resale value of the $t = 1$ investment βI now depends on the state of nature. Concretely, we assume that in the good state all investment costs (βI) can be recovered, in the bad state only half of the investment costs can be regained ($1/2\beta I$). The aim of this analysis is to show that paradoxical effects still occur in this modified setting.

For illustrative purposes and sake of convenience, we only consider the case in which the $t = 1$ investment is abandoned in the good state but not in the bad state.

$$\frac{1}{2}\left[(1-\tau)\,\alpha\frac{CF_0+1}{1+r_{\tau_f}} - \beta\frac{I}{1+r_{\tau_f}}\right] + \frac{1}{2}\left[-\frac{1}{2}\beta\frac{I}{1+r_{\tau_f}}\right] \tag{27}$$

Hence, a necessary criterion for delaying the project is that

$$\frac{1}{2}\left[(1-\tau)\,\alpha\frac{CF_0+1}{1+r_{\tau_f}} - \frac{3}{2}\beta\frac{I}{1+r_{\tau_f}}\right] \geq (1-\tau)CF_0 - I. \tag{28}$$

This condition can be rewritten as

$$I\left(1 - \frac{3}{4}\frac{\beta}{1+r_{\tau_f}}\right) + \frac{1}{2}\frac{1-\tau}{1+r_{\tau_f}}\alpha \geq (1-\tau)CF_0\left(1 - \frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right). \tag{29}$$

As before, noting that $\alpha < 1 + r_{\tau_f}$, this leads to a cut-off level CF_0^* that is given by

$$CF_0^* = \max\left\{0, \frac{I\left(1 - \frac{3}{4}\frac{\beta}{1+r_{\tau_f}}\right)}{(1-\tau)\left(1 - \frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)} + \frac{1}{2}\frac{\frac{\alpha}{1+r_{\tau_f}}}{\left(1 - \frac{1}{2}\frac{\alpha}{1+r_{\tau_f}}\right)}\right\} \tag{30}$$

and the investment is delayed for all $CF_0 \leq CF_0^*$. Again, since $\beta > 1 + r_{\tau_f}$ and $\alpha > 2(1 + r_{\tau_f})$ cut-off level CF_0^* is decreasing in τ as long as CF^* is strictly positive. As before, this proves that paradoxical tax effects occur in the modified setting.

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