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How Much to Pay for Tax Certainty? The Role of Advance Tax Rulings for Risky Investment under Loss Offset and Tax Uncertainty

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Abstract

This study examines the impact of tax certainty through advance tax rulings (ATRs) on firms' risky investments under cash flow and tax uncertainty. Both firms and governments have expressed growing concern about increasing tax uncertainty, due to frequent tax reforms and the difficulty in applying ambiguous tax laws and anticipating audit outcomes. One remedy is the provision of ATRs, which offer upfront clarification of tax issues to reduce tax uncertainty and increase risk-taking. We analyze how these uncertainties, along with different tax rates, loss offset provisions, and ATR fees affect investment strategies. Our results suggest, first, that ATRs encourage riskier investments, particularly in tax regimes with generous loss offsets. We identify optimal ranges of ATR fees that benefit firms and tax authorities. Second, we show that it may be beneficial to design ATRs with a low or negative fee. Third, our study reveals a U-shaped relationship between firms' risk aversion and their willingness to pay for tax certainty, with willingness being higher for firms at low or high levels of risk aversion. In contrast, moderately risk-averse firms are only willing to accept low fees. Overall, our results highlight the importance of low-cost tax certainty combined with generous loss offset provisions to encourage risky investments.

Keywords: tax uncertainty, advance tax ruling, cash flow uncertainty, loss offset, optimal investment, willingness to pay.*

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1 Introduction

This study investigates how firms' willingness to pay for advance tax rulings (ATRs) and tax authorities' willingness to offer them affects firms' investment decisions under both tax and cash flow uncertainty. Tax policy is a central tool for encouraging risky investments (IMF 2017). However, incentives, such as lower tax rates or generous loss offsets that encourage risk-taking and investment (Ljungqvist et al. 2017; Langenmayr and Lester 2018; Bethmann et al. 2018; Dharmapala 2024), might be undermined by tax uncertainty (Osswald and Sureth-Sloane 2024).

Tax uncertainty arises from vague legislation, frequent reforms, and discretion in audits, which complicate firms' ability to assess after-tax returns (Mills et al. 2010; Lisowsky et al. 2013; Neuman et al. 2020). Policymakers and practitioners increasingly recognize tax uncertainty as a barrier to investment (IMF and OECD 2019; Jacob et al. 2022; Fox et al. 2022; Gallemore et al. 2025). Anecdotal evidence gathered through informal conversations with practitioners and interviews highlights the problem. For example:

> *... board members [...] not only [...] look at [...] where costumers are and where to produce cheaply, but [...] also [consider] other risks, and that definitely includes taxes, where we [tax department] are always involved. I have two cases where [we] are even going to leave [the country] because the tax risks are too high.*
>
> —Head of Taxes, public European manufacturer.

This barrier is particularly problematic for high-risk projects, as it compounds cash flow uncertainty. To address this uncertainty, many jurisdictions offer ATRs—binding, upfront guidance on the tax treatment of planned transactions. While ATRs are widely regarded as a remedy (IMF and OECD 2018; Hoppe et al. 2023), their economic effects are not well understood. Studies (Diller et al. 2017; Becker et al. 2017) abstract from the interaction of tax and cash flow uncertainty and the asymmetric tax treatment of profits and losses, neglecting important features of risky investment.

This study fills this gap by modeling tax uncertainty as uncertainty in tax payments arising from ambiguous treatment of both the tax base and asymmetric tax rates on profits and losses.[1] This issue is particularly relevant for R&D investments, which are uncertain in both outcomes and tax treatment and often generate intangibles that are hard to value and prone to tax disputes. Recent tax reforms, such as those under the OECD's BEPS project (OECD 2013), including the nexus and DEMPE frameworks, have further aggravated complexity and tax uncertainty. Real-world examples, such as Facebook's dispute with the U.S. IRS involving nearly $9 billion in taxes (White 2020), illustrate the material risk involved. Fiscal pressure following the COVID-19 pandemic, combined with the need to finance climate-transition and defense programs, are likely to fuel tax disputes and further increase tax uncertainty (KPMG 2016; Nessa et al. 2020).

[1] This includes uncertainty from reforms, vague definitions, and auditor discretion.

As tax uncertainty grows, ATRs become more valuable. Although most OECD countries offer them (OECD 2019), their availability, scope, and pricing differ significantly across jurisdictions. Some are free , while others entail fees based on such factors as time spent, complexity, or the monetary stakes involved.[2] Firms must weigh these costs against the potential benefits when making investment decisions under uncertainty.

We develop a decision model that incorporates both cash flow uncertainty and tax uncertainty to analyze how ATRs affect firms' investments and how tax authorities can optimally design ATR fees. Building on Diller et al. (2017), we allow for interaction between tax rate uncertainty, cash flow uncertainty, and the asymmetric treatment of losses, particularly in the context of risky projects, such as R&D investments. Our analysis focuses on two key questions: (i) how does access to ATRs affect a firm's optimal investment strategy under tax uncertainty, and (ii) what constitutes an efficient ATR fee that balances the interests of both the tax authority and the taxpayer?

Our model yields three main findings. First, consistent with Diller et al. (2017), we find that ATRs do encourage investment in risky projects. Furthermore, we show that this effect strengthens when combined with generous loss offset provisions. Without certainty instruments like ATRs, the investment-stimulating effect of loss offsets does not materialize, especially in high-tax environments.

Second, we show that, under high tax uncertainty, offering ATRs at low or even negative fees—effectively subsidizing certainty—is often necessary to stimulate investment. While earlier research identifies only nonnegative fee thresholds, our results suggest that, under specific circumstances, such as when tax rates are high, compliance cost savings or investment incentives are substantial, it is optimal for tax authorities to subsidize the provision of tax certainty.

Third, we find a U-shaped relationship between firms' willingness to pay for ATRs and their degree of risk aversion. Firms with low risk aversion value certainty to secure high expected returns, while those with high risk aversion are especially sensitive to residual uncertainty and may be willing to pay a premium for tax certainty. Firms with moderate risk aversion, in contrast, are least willing to pay for ATRs, as neither the investment incentive nor their aversion to uncertainty is sufficiently strong to justify a high fee. This U-shaped pattern arises from the interaction of opposing forces: (1) investment-driven demand for tax certainty, which dominates at low risk aversion, due to high stakes in risky investments, and (2) uncertainty avoidance, which dominates at high risk aversion, due to a preference for predictability across all kinds of investment decisions. At intermediate levels of risk aversion, neither force is strong enough to justify paying a high fee. This leads to a dip in the critical fee that firms are willing to pay.

In sum, our findings position ATRs as a strategic policy tool that can mitigate tax-induced investment frictions and encourage risk-taking while enabling tax authorities to manage the trade-off between providing certainty and securing revenue.

[2]See, e.g., Internal Revenue Service (2016); OECD (2019); Canada Revenue Agency (2022); South African Revenue Service (2023).

Our study contributes to the literature on investment decisions under uncertainty (e.g., Niemann and Sureth 2004; Alvarez and Koskela 2008; Gries et al. 2012; Kanniainen and Panteghini 2013; Agliardi 2001). While research demonstrates that tax uncertainty can deter investment (Sialm 2006; Niemann 2011; Fox et al. 2022; Lee and Shevlin 2023), it largely overlooks how to overcome this barrier, such as ATRs. Some studies highlight potential downsides of tax certainty instruments, such as strategic disadvantages or heightened scrutiny from tax authorities (Givati 2009), while others examine cooperative compliance programs or advance pricing agreements but focus solely on their effects on audit and compliance costs (De Simone et al. 2013; Becker et al. 2017). These studies do not explore how tax uncertainty shields affect firms' investments. A notable exception is Diller et al. (2017), who analyze the value of ATRs under tax uncertainty. Yet their model abstracts from key features of investment decisions, such as stochastic cash flows, loss asymmetries, and firm risk preferences.

Our model integrates tax rate uncertainty, cash flow risk, asymmetric loss treatment, and firm risk aversion, allowing us to assess how ATRs influence risky investment decisions and to derive fee structures that balance the interests of both taxpayers and tax authorities. Our analysis provides novel insights into how ATRs, especially when combined with generous loss offset provisions, can stimulate investment, even in high-tax environments among risk-averse firms. Our results also advance optimal asset allocation theory by incorporating crucial tax frictions and providing closed-form solutions that account for the interaction of tax and cash flow uncertainty.

From a policy perspective, our findings provide guidance for designing ATR fee schemes that can stimulate investment without compromising fiscal objectives. Specifically, our results show that low-cost or even subsidized tax certainty instruments—such as ATRs or cooperative compliance programs—can help encourage firms to take risk in environments with high tax uncertainty. Importantly, the benefits of ATRs are amplified when combined with generous tax loss offsets. Policymakers aiming to promote risk-taking and innovation should therefore prioritize the expansion and accessibility of tax certainty instruments, ensuring that their fees and integration with loss offsets are calibrated to firms' risk profiles and the prevailing level of tax uncertainty.

We encourage future research to empirically test our theoretical predictions and to evaluate the practical effectiveness of different tax certainty instruments across jurisdictions. In particular, further evidence is needed on how firms respond to varying ATR designs and pricing models in conjunction with loss offset regimes.

2 Model setup, investment strategy, and tax procedures

We introduce the investment problem and the underlying financial market under two alternative tax procedures, that is, with and without an ATR. In a first step, an investor (a firm) wants to decide how much to invest in a risky project such as R&D. We solve for the optimal investment amount from the firm's perspective in both settings. In a second step, we determine reasonable fee ranges for the ATR and thereby incorporate the tax authority's decision on the ATR fee, which anticipates the firm's decision-making.

2.1 Optimal investment amount

We consider a time horizon $[0, T]$, $T < \infty$. The firm is endowed with an initial wealth X_0 and can invest in (1) a risky investment project (e.g., R&D) that yields a normally distributed T-year return $R_T \sim \mathcal{N}(\mu T, \sigma^2 T)$, where $\mu > 0$, the rate of return on the risky project, and $\sigma > 0$, the volatility, are constants and (2) a risk-free government bond that accrues at a constant annual interest rate r. We assume that the expected rate of return on the risky project exceeds the risk-free rate r; i.e., $\mu - r > 0$. The risk-free rate r can be interpreted as the deterministic post-tax rate. We use this simple model to allow for analytical tractability of the optimal asset allocation problem. We denote the amount invested in the risky project by $\theta \in [0, X_0]$. Consequently, the firm's time-T wealth is given by

$$X_T = \theta \cdot (1 + R_T) + (X_0 - \theta) \cdot (1 + r \cdot T) \,. \tag{1}$$

The parameter θ steers the riskiness of the investment and is later the choice variable. This approach implies that the firm's time-T wealth X_T is normally distributed with mean and variance

$$\mathbb{E}[X_T] = \theta \cdot (1 + \mu \cdot T) + (X_0 - \theta) \cdot (1 + r \cdot T) \,, \tag{2}$$

$$\mathrm{Var}[X_T] = \sigma^2 \theta^2 T \,. \tag{3}$$

Taxation and ATR

We assume that the net payoff from the investments, $X_T - X_0$, may be positive (profit) or negative (loss) and is subject to different stochastic proportional tax rates, depending on its profit or loss character. Here and in the following, the term "net payoff" denotes the terminal wealth less the initial investment.

If $X_T - X_0$ is positive, a profit tax rate $\tilde{\tau}_p$ is applied, leading to an after-tax payment of $(1-\tilde{\tau}_p)(X_T - X_0)$.[3] Assuming that the taxation of the net payoff from the risky project is subject to tax uncertainty, we describe $\tilde{\tau}_p$ as a binary random variable

$$\tilde{\tau}_p = \begin{cases} \delta_p \cdot \tau & \text{with probability } d\,, \\ \tau & \text{with probability } 1-d\,, \end{cases} \tag{4}$$

where $d \in (0,1)$ is the probability of an adjustment of the assessed or anticipated tax payment, due to a tax audit or reform. This adjustment translates into an increase in the tax rate τ by a tax risk multiplier $\delta_p \in (1, \frac{1}{\tau})$. For simplicity, we focus in our interpretations on tax audits as possible causes for such a tax adjustment. However, our analysis can be easily generalized to tax adjustments due to tax reforms.

Typically, audit probabilities vary considerably within and across countries (Mendoza et al. 2017; Bachas et al. 2019). A higher probability of a tax audit indicates higher tax uncertainty. We analogously introduce a stochastic proportional tax rate for losses $\tilde{\tau}_l$ if the net payoff $X_T - X_0$ is negative. We obtain an after-tax payment of $(1-\tilde{\tau}_l)(X_T - X_0)$. We describe $\tilde{\tau}_l$ as a binary random variable with δ_l the tax risk multiplier for losses

$$\tilde{\tau}_l = \begin{cases} \delta_l \cdot \lambda \cdot \tau & \text{with probability } d\,, \\ \lambda \cdot \tau & \text{with probability } 1-d\,. \end{cases} \tag{5}$$

Here, the tax loss offset parameter $\lambda \in [0,1]$ captures the asymmetric nature of taxation of profits and losses resulting from loss offset restrictions. The loss offset parameter λ takes the value $\lambda = 1$ if losses can be completely and immediately offset, implying an immediate tax refund at the tax rate τ (symmetric taxation of profits and losses). If the loss offset is restricted ($\lambda < 1$), unused losses can be carried forward until future profits allow for an offset or expire for tax loss offset purposes. If $\lambda = 0$, the tax loss cannot be offset at all. With probability $d \in (0,1)$, a tax audit leads to a post-tax audit reduction of the tax offset by a tax risk multiplier $\delta_l \in (0,1)$.[4]

With $\tilde{\tau}_p$ and $\tilde{\tau}_l$ and the parameters d, δ_p and δ_l, we operationalize tax uncertainty from a tax audit that might lead to a higher post-audit tax burden than originally assessed.[5] We summarize the after-tax payoff absent an ATR in Procedure 1:

[3] We abstract from tax-deductible accruals.

[4] A reduction in tax loss offset might be due to a declared tax expenses that are not considered tax-deductible by the tax authority. The tax risk multiplier for losses δ_l can also be interpreted as a parameter that reflects the risk of having insufficient future cash flows for the loss offset.

[5] This definition of tax uncertainty is narrower than overall tax uncertainty. Hence our results reflect the lower bound of tax uncertainty, as uncertainty can stem from other sources, too.

Procedure 1 (no ATR): If the company decides not to request an ATR, the after-tax net payoff is given by

$$\tilde{X}_T^{(1)} = \begin{cases} (1-\tilde{\tau}_p)(X_T - X_0), & \text{if } X_T \geq X_0\,, \\ (1-\tilde{\tau}_l)(X_T - X_0), & \text{if } X_T < X_0\,. \end{cases} \tag{6}$$

By assumption and the introduction of the random tax rates $\tilde{\tau}_p$ and $\tilde{\tau}_l$ in equations (4) and (5), respectively, we observe that the tax audit leads to a lower after-tax net payoff $\tilde{X}_T^{(1)}$. In case of profits ($X_T \geq X_0$), with probability d, the firm must pay a higher tax at rate $\tau \cdot \delta_p > \tau$, and with probability $1-d$, the project's tax rate is τ. In case of a loss ($X_T < X_0$), with probability d, the firm receives a tax refund at a lower rate $\delta_l \cdot \lambda \cdot \tau < \lambda \cdot \tau$ on tax losses, and, with probability $1-d$, the firm retains a tax loss offset at rate $\lambda \cdot \tau$. □

The firm can request an ATR, which protects against tax uncertainty. After paying a fee F_0, the ATR ensures that the tax in the loss and profit domain are fixed. Thus, the random tax rate $\tilde{\tau}_p$ is replaced by a deterministic tax rate $\eta_p \cdot \tau$, and tax uncertainty is resolved. The tax rate multiplier for profits η_p is $\in (1, \delta_p)$ and reflects the increase in the tax rate as a consequence of the tax certainty provided to the firm by the ATR. This rate can be chosen by the tax authority such that the resulting combined tax rate lies between the two random outcomes of $\tilde{\tau}_p$ as described in equation (4). Correspondingly, the ATR also fixes the tax rate in the case of a loss at the level of η_l with $\lambda \cdot \tau$. Here $\eta_l \in (\delta_l, 1)$ is the tax rate multiplier for losses set by the tax authorities in the ATR such that the resulting deterministic combined tax rate lies in the interval as described by equation (5). The firm can choose whether it requests this ATR and pays the fee F_0 or whether the project's tax uncertainty remains. This decision depends on the tax rate multipliers η_p, η_l and the ATR fee F_0. Later, when we discuss reasonable choices for the ATR fee F_0, we will elaborate on the relationship between η_p, η_l, and F_0.

We summarize the after-tax payoff of Procedure 2 with ATR:

Procedure 2 (ATR): The firm decides to initiate an ATR by paying a fixed fee F_0 upfront. The aim of initializing an ATR is to avoid the tax uncertainty related to the random tax rates $\tilde{\tau}_p$ and $\tilde{\tau}_l$ for profits and losses, respectively. Hence, with an ATR, the firm pays for a fixed tax rate, which, however, can differ from τ. We assume that the ATR is perfect, i.e., that it eliminates tax uncertainty. The firm can therefore replace its random tax rates ($\tilde{\tau}_p$, $\tilde{\tau}_l$) with fixed deterministic tax rates $(\eta_p \cdot \tau, \eta_l \cdot \tau)$. The after-tax net payoff of the company at time T is as follows:

$$\tilde{X}_T^{(2)} = \begin{cases} (1-\eta_p \cdot \tau)(X_T - X_0) - F, & \text{if } X_T \geq X_0\,, \\ (1-\eta_l \cdot \tau)(X_T - X_0) - F, & \text{if } X_T < X_0\,, \end{cases} \tag{7}$$

where we accrue the ATR fee F_0 at the risk-free rate r to obtain $F := F_0(1 + r \cdot T)$ at time T. For solvability reasons, we assume a nontax-deductible fee. Explicitly modeling a tax-deductible fee does not allow for closed-form solutions. Therefore, we implicitly assume F_0 being an after-tax fee. Note that the after-tax net payoff in the profit case $X_T \geq X_0$ can be negative after deducting the fee F_0. Intuitively, in this case, the firm has no incentives to invest in the project with an ATR. We will elaborate on this when we determine the critical fee level where the firm is indifferent between Procedures 1 (no ATR) and 2 (ATR).

Optimization

To determine the maximal expected utility of the firm for the two payoff schemes for a given fee, we assume that the firm's preferences can be described by an exponential utility function; that is, $U(X) = -\frac{1}{\gamma}e^{-\gamma X}$ with a risk-aversion coefficient $\gamma > 0$. We assume risk aversion for firms in investment decisions, as it is well supported by both theoretical and empirical work in corporate finance (Merton 1969; Merton 1971; Basak and Shapiro 2001; Chen et al. 2011). Firms face multiple sources of uncertainty—not only in the returns on their investments (such as R&D) but also in external factors like market volatility, economic shocks, and regulatory or tax uncertainties. Managers derive their utility from firms' returns through performance-linked compensation (Hall and Murphy 2002; Coles et al. 2006), making them potentially risk-averse and inclined to avoid high-risk projects. Modeling firms as risk averse hence allows us to capture the fact that decision-makers often weigh potential losses more heavily than gains, leading to more conservative investment policies when uncertainty is high. Through the incorporation of risk aversion, we extend Diller et al. (2017), where a firm's decisions are based on expected after-tax cash flows. By contrast, we investigate the attractiveness of ATRs, depending on the firm's risk aversion. The employed exponential utility is widely used and justified by work in economics, finance, and insurance and risk management; see, for example, Carmona (2009). Because in our model the after-tax net payoff can be negative, we cannot describe preferences by logarithmic or power utility that are exclusively defined for the positive real line.

The firm wants to maximize the expected utility from the after-tax net payoff. The optimal investment problem of the firm under Procedures 1 and 2 is then given by

$$\max_{\theta \in [0, X_0]} EU^{(i)}(\theta) := \max_{\theta \in [0, X_0]} \mathbb{E}\left[-\frac{1}{\gamma} \exp\left\{-\gamma \tilde{X}_T^{(i)}\right\}\right], \; i = 1, 2\,, \tag{8}$$

$$\text{s.t. } X_T \text{ and } \tilde{X}_T^{(i)} \text{ given by (1), (6), (7).}$$

Using this objective function, we are assuming that the firm aims to maximize excess wealth, where the initial wealth X_0 is chosen as a basis for comparison. Note that maximizing the excess wealth also relates to minimizing the probability that terminal wealth X_T falls below the initial wealth level X_0. The lower this probability, the higher the expected utility resulting from the excess wealth.

Note that the ATR is not a variable that can be chosen at will but results from a two-step approach. Initially, we select one of the two available tax procedures and examine the optimal investment scenario based on this selection. We subsequently repeat this optimization process for the alternative tax procedure. In the second step, we compare the outcomes of both procedures to determine the conditions under which applying the ATR becomes optimal.

For mathematical tractability, we abstract from the fact that tax rates on profits from risky investments like R&D might differ from tax rates on interest income.[6]

Let us first discuss the special case of an ATR with *symmetric taxation.* ($\eta := \eta_p = \eta_l = 1$, $\lambda = 1$). Here, the optimal investment decision under exponential utility is well studied; see, for example, Seifried (2010).

Proposition 1 (Optimal investment amount, symmetric taxation, ATR)**.** *If the net payoff $X_T - X_0$ is subject to a symmetric tax $\eta\tau$ under Procedure 2 (ATR), that is, if $\eta := \eta_p = \eta_l = 1$, $\lambda = 1$, the optimization problem* (8) *admits a closed-form solution given by:*

$$\theta^s = \min\left\{\frac{\mu - r}{\gamma\sigma^2(1-\tau)},\, X_0\right\} \tag{9}$$

Proof. Abbreviating $\tilde{\mu} := (\mu - r)T$ and $\tilde{\sigma} := \sigma\sqrt{T}$, we find for a constant symmetric tax τ that

$$\mathbb{E}\left[-\frac{1}{\gamma}\exp\{-\gamma(1-\tau)(X_T - X_0)\}\right] = -\frac{\text{const.}}{\gamma}\exp\left\{-\gamma(1-\tau)\tilde{\mu}\theta + \frac{1}{2}\gamma^2(1-\tau)^2\tilde{\sigma}^2\theta^2\right\}.$$

Taking first-order conditions with respect to θ, we find that this objective is maximized if and only if the maximizer θ^s is given by equation (9). □

The rationale behind Proposition 1 is that an increased expected tax burden (represented by a higher τ) corresponds to a greater optimal allocation to risky investments, denoted as θ^s.[7] Further, the optimal investment amount θ^s reflects the trade-off between the desire to invest based on expected returns and risk (captured by the term $\frac{\mu-r}{\gamma\sigma^2(1-\tau)}$) and practical constraints on available capital X_0. The risk aversion parameter γ plays a crucial role in determining the investment amount. In particular, for a risk aversion $\gamma < \frac{\mu-r}{X_0\sigma^2(1-\tau)}$, the term $\frac{\mu-r}{\gamma\sigma^2(1-\tau)}$ exceeds X_0, in which case the investment is capped at X_0, ensuring the investor does not exceed the available resources. From this,

[6] Accordingly, we also abstract from the resulting implications for the after-tax net payoff of the firm under such a tax system (e.g., a tax system with a lower proportional tax rate on interest income). However, in Appendix B, we show how the after-tax net payoff of the firm can be modified to this more granular tax framework.

[7] This proposition is consistent with the well-known paradigm for symmetric taxation of profits and losses (Domar and Musgrave 1944).

we can conclude that the optimal investment may be close in magnitude to the initial investment, depending on the level of risk aversion.

Note that the rationale behind Proposition 1 applies to situations where the tax rate is deterministic and well defined, such as in the case of an ATR. To explore how the ATR influences the investment decision, we shall now consider the more general case of *asymmetric taxation*.

First, allowing for differential tax multipliers provides the government with additional flexibility to fine-tune the tax burden on profits versus losses. In practice, tax authorities often treat profits and losses differently. By adjusting η_p and η_l separately, the government can mitigate the distortions that arise from tax uncertainty in a more targeted way. For example, firms might be particularly sensitive to increases in the tax rate on profits (which directly erodes gains), while the effect of altering loss offsets might differ, due to their role in smoothing tax liabilities over time. Second, the additional degrees of freedom facilitate achieving revenue neutrality and balancing incentives more precisely. In some settings, aligning the deterministic tax rate with the simple expected value might not fully capture the strategic responses of firms to asymmetric taxation or the effects of loss offset restrictions. Our framework thus captures richer dynamics, as it allows the policy mix (η_p, η_l, λ, F) to be calibrated to address these issues simultaneously.

To disentangle these interactions, in the following, we examine, first, the case without ATR and thus no corresponding interaction between uncertainty in the post-audit tax rate and the optimal investment decision. (See Proposition 2 and the analysis at the end of Section 3.) Second, we investigate the case of a loss-offset restriction ($\lambda < 1$) but a deterministic tax rate under an ATR. (See Proposition 3 and the analysis in Section 3.) In both cases, the optimization problem can still be solved semi-analytically.

To address the optimization problem, we again employ a two-step approach. First, for a given investment amount, we calculate the expected utility of the firm. Subsequently, we derive the optimal investment amount for the risky project under both tax procedures, considering scenarios with and without the ATR. You can find comprehensive derivations in Appendix B.

Proposition 2 (Expected utility, optimal investment, asymmetric taxation, no ATR)**.** *Under Procedure 1 (no ATR), the expected utility as a function of the investment amount $\theta \in [0, X_0]$ is given by*

$$\begin{aligned}
EU^{(1)}(\theta) &= -\frac{1}{\gamma}\mathbb{E}\left[\exp\{-\gamma \tilde{X}_T^{(1)}\}\right] \\
&= -\frac{d}{\gamma}\big(\exp\{\gamma(1-\delta_p\tau)X_0\}\cdot g_1(X_0,\tau\delta_p) + \exp\{\gamma(1-\lambda\tau\delta_l)X_0\}\cdot g_2(X_0,\lambda\tau\delta_l)\big) \\
&\quad - \frac{1-d}{\gamma}\big(\exp\{\gamma(1-\tau)X_0\}\cdot g_1(X_0,\tau) + \exp\{\gamma(1-\lambda\tau)X_0\}\cdot g_2(X_0,\lambda\tau)\big),
\end{aligned}$$

where $g_1(a,b)$ and $g_2(a,b)$ are given by

$$\begin{aligned} g_1(a,b) &:= \mathbb{E}\big[\exp\{-\gamma(1-b)X_T\}\cdot \mathbb{1}_{\{X_T\geq a\}}\big] \\ &= \exp\left\{-\gamma(1-b)\mathbb{E}[X_T]+\frac{1}{2}\gamma^2(1-b)^2\operatorname{Var}[X_T]\right\} \\ &\qquad\cdot\Phi\left(-\frac{a-\mathbb{E}[X_T]}{\sqrt{\operatorname{Var}[X_T]}}-\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right), \qquad (10) \\ g_2(a,b) &:= \mathbb{E}\big[\exp\{-\gamma(1-b)X_T\}\cdot \mathbb{1}_{\{X_T< a\}}\big] \\ &= \exp\left\{-\gamma(1-b)\mathbb{E}[X_T]+\frac{1}{2}\gamma^2(1-b)^2\operatorname{Var}[X_T]\right\} \\ &\qquad\cdot\Phi\left(\frac{a-\mathbb{E}[X_T]}{\sqrt{\operatorname{Var}[X_T]}}+\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right). \qquad (11) \end{aligned}$$

The optimal amount $\theta^* \in [0, X_0]$ solving equation (8) for Procedure $i = 1$ is either 0, X_0 or determined implicitly and uniquely by solving

$$\frac{\partial EU^{(1)}(\theta)}{\partial\theta} = 0\,. \qquad (12)$$

Proof. See Appendix B. We also provide an analytic expression for $\frac{\partial EU^{(1)}(\theta)}{\partial\theta}$. □

With Proposition 2, we derive an analytical expression for the expected utility of the firm as a function of the investment amount θ under the first tax procedure without an ATR. This proposition examines the scenario in which the firm faces asymmetric taxation without the option of an ATR. In this setting, tax rates applied to profits and losses are uncertain because they depend on whether a tax audit occurs. Specifically, when the firm's net payoff is positive, it may be taxed at a higher rate with a certain probability due to an audit (captured by the multiplier δ_p), while in the case of losses, the tax benefit is similarly uncertain because an audit may reduce the loss offset (through the multiplier δ_l). This dual structure introduces an additional layer of risk to the investment decision. The four terms of the expected utility function derived in Proposition 2 reflect the four possible regimes: profit and ATR, profit without ATR, loss with ATR, and profit with ATR. In case there is no loss offset and taxes for gains and losses are equal, the four terms reduce to a single term

$$\mathbb{E}\left[-\frac{1}{\gamma}\exp\left\{-\gamma(1-\tau)(X_T-X_0)\right\}\right]$$

from Proposition 2. In the numerical section, we will show that the four terms lead to some nontrivial relations between optimal investment, cash flow, and tax uncertainty.

Proposition 2 shows that the firm's optimal investment decision is determined by maximizing the expected utility that now includes the stochastic elements of tax outcomes. Since the tax rates can jump to higher or lower levels, depending on audit outcomes, the decision on how much to invest in the risky project becomes a balancing act. On the one hand, the firm is attracted to the higher potential gains from the risky investment; on the other, it must weigh these against the possibility of an adverse tax adjustment, which would reduce the net benefit of the investment. As a result, the optimal investment amount, denoted as θ^*, is found by setting the derivative of the expected utility with respect to the investment level to zero. This implicitly defined optimum captures how the firm's risk aversion and the uncertainty of tax treatment interact to influence its overall risk-taking.

Proposition 3 derives the analogous expression for the tax procedure with an ATR. Since the ATR leads to the application of a fixed and therefore deterministic tax rate, computing the firm's expected utility is simpler than in the presence of tax uncertainty. In Procedure 2, the fee F_0 has a significant impact on the magnitude of the expected utility. However, it does not affect the optimal investment amount θ^{**} under the ATR tax procedure.[8]

Proposition 3 (Expected utility, optimal investment, asymmetric taxation, ATR)**.** *Under Procedure 2 (ATR), the expected utility as a function of the investment amount $\theta \in [0, X_0]$ is given by*

$$\begin{aligned} EU^{(2)}(\theta) &= -\frac{1}{\gamma}\mathbb{E}\left[\exp\{-\gamma \tilde{X}_T^{(2)}\}\right] \\ &= -\frac{e^{\gamma F}}{\gamma}\Big(\exp\{\gamma(1-\tau\eta_p)X_0\}\cdot g_1(X_0, \tau\eta_p) + \exp\{\gamma(1-\lambda\tau\eta_l)X_0\}\cdot g_2(X_0, \lambda\tau\eta_l)\Big)\,, \end{aligned}$$

where $F := F_0(1+r\cdot T)$ and $g_1(a,b)$ and $g_2(a,b)$ are defined as in Proposition 2. The optimal amount $\theta^{**} \in [0, X_0]$ solving equation (8) for Procedure $i = 2$ is either 0, X_0 or determined implicitly and uniquely solving

$$\frac{\partial EU^{(2)}(\theta)}{\partial \theta} = 0\,. \tag{13}$$

Proof. See Appendix B. We also provide an analytic expression for $\frac{\partial EU^{(2)}(\theta)}{\partial \theta}$. □

Proposition 3 considers an alternative scenario where the firm chooses to pursue an ATR. With an ATR, it pays an upfront fee to replace the stochastic tax rates with fixed, deterministic ones. In this case, the tax rate for profits is set to $\eta_p\tau$ and for losses to $\eta_l\tau$, removing any uncertainty

[8] This finding resembles the result observed in optimal insurance contracts. If an actuarially fair premium is charged for an insurance contract, the optimal insurance purchase of a risk-averse policyholder is full insurance. If instead a premium with a fixed surcharge is applied, the optimal insurance purchase of a risk-averse policyholder is either still full insurance or no insurance (which is the case if the surcharge is too high).

associated with tax audits. The resulting expected utility function under the ATR is more straightforward, as it no longer needs to accommodate the randomness of the tax rates. However, the fee paid for the ATR, which grows at the risk-free rate to its terminal value F, directly reduces the firm's net payoff. Therefore, while the ATR provides certainty and can be seen as a form of insurance against tax risk, its cost must be carefully weighed against the benefits of eliminating uncertainty.

The economic intuition behind Proposition 3 is that, by removing tax uncertainty, the firm may be able to make a more efficient investment decision, better aligning its chosen level of investment with its risk-return trade-off. In a deterministic tax environment, the firm's optimal investment amount (denoted as θ^{**}) is derived by solving a simpler optimization problem, similar to that under symmetric taxation but adjusted for the fee. The ATR effectively shifts the risk profile of the investment: Although the risk of unexpectedly high taxes on profits or diminished tax benefits on losses is eliminated, the firm incurs a cost by paying the fee, which reduces its overall profitability. This trade-off is central to the decision-making, and the optimal investment level will reflect the balance between the certainty provided by the ATR and the burden imposed by its fee.

Together, these propositions illuminate the critical role that tax uncertainty and its mitigation play in a firm's investment decision. Without an ATR, the firm faces a more complex environment where the combination of cash flow and tax uncertainty leads to a careful, risk-adjusted investment strategy. With an ATR, the trade-off shifts. Tax certainty is achieved at a price, potentially allowing the firm to invest more confidently if the fee is sufficiently low relative to the risk reduction. Both cases underscore the importance of incorporating tax uncertainty considerations into investment decisions, demonstrating how variations in tax policy and enforcement can materially affect optimal risk-taking.

In both Propositions 2 and 3, note that simulation is not required to determine the optimal investment amounts. Since the expected utility is given in closed form, the optimal investment (and subsequently the indifference fee) can be obtained by numerically solving a fixed-point problem (12), respectively (13), implementing a root search algorithm. Although a closed-form solution for the optimal investment amount does not exist, we can still anticipate certain parameter effects. Theorem 4 shows that the tax loss offset parameter λ has a monotonic influence on the optimal investment amount θ^{**}.

Theorem 4 (Impact of the tax loss offset parameter λ)**.**
Assume that $\frac{\partial g_2(X_0,\lambda\tau\eta_l)}{\partial\theta} > 0$.

(a) If the amount θ is left unchanged, the expected utility $EU^{(2)}(\theta)$ is increasing in the tax loss offset parameter λ.

(b) An increase (decrease) in the tax loss offset parameter λ leads to an increase (decrease) in the optimal amount θ^{**}. This further increases $EU^{(2)}(\theta)$ from (a).

Proof. See the Appendix B, 4. □

In the ATR framework of Proposition 3, the loss offset parameter λ plays a critical role in shaping the firm's effective risk profile in the loss domain. Recall that λ represents the fraction of the tax loss that the firm can immediately offset when facing losses.

When $\lambda = 1$, the firm enjoys full tax relief for its losses, whereas a lower λ implies that the loss relief is restricted. In the expected utility expression for the ATR procedure, λ enters only the term corresponding to the loss scenario. A higher λ improves the after-tax outcome in the case of losses because it allows the firm to recover more of its lost amount through tax refunds. This, in effect, lowers the downside risk associated with the risky investment. Since the optimal investment decision θ^{**} is derived from maximizing the overall expected utility, any factor that reduces downside risk generally makes the investment more attractive. With a more generous loss offset (i.e., a higher λ), the negative impact of losses on the firm's wealth is mitigated, which in turn reduces the firm's effective risk aversion. In a typical risk-return trade-off, a reduction in downside risk encourages a risk-averse investor to allocate more to the risky asset. Therefore, all else being equal, an increase in λ results in a higher optimal investment amount θ^{**}; see Theorem 4.

This relationship is monotone in nature. As λ increases from a lower value (indicating stricter loss offset restrictions) toward 1 (full loss offset), the improvement in the after-tax payoff for the loss branch is systematic and continuous. Consequently, the optimal investment θ^{**} adjusts upward monotonically as the downside protection improves. In summary, within the ATR setting of Proposition 3, a higher loss offset parameter λ monotonically increases the optimal investment in the risky project by reducing effective downside risk and enhancing the firm's risk profile.

2.2 Pareto-improving fee range

So far, we have determined the optimal investment amount under the two different tax procedures, assuming that the ATR fee level is given (the second step of our approach). In the following, we analyze how this fee can be reasonably set, considering the viewpoint of both the firm and the tax authority. We determine the firm's maximal willingness to pay for the ATR, denoted by F_0^*, and the lowest fee that is acceptable to the tax authority, denoted by F_0^{**}. If $F_0^* < F_0^{**}$, an ATR will not be requested by the firm. Thus ATR fees with $F_0 \in [F_0^{**}, F_0^*]$, for $F_0^* \geq F_0^{**}$, constitute a Pareto-improving fee range that benefits both the firm and the tax authority.

Firm's point of view

Whether the firm is willing to request an ATR depends on the firm's risk preferences, the tax system and tax uncertainty parameters, and the ATR fee in both tax procedures (i.e., the parameters d, δ_p, δ_l, λ, τ). In what follows, we assume that the model parameters are given, except the ATR fee F_0. We further assume that the firm always chooses the optimal investment amount θ^* (optimal amount under tax procedure 1) and θ^{**} (optimal amount under tax procedure 2) respectively. We determine the critical fee F_0^* that leads to identical utility levels for both tax procedures for the firm (the firm's procedural indifference). The firm will request an ATR only if the expected utility is at least as high as absent an ATR. As the utility of the firm decreases monotonically with the fee, we conclude that the firm will request the ATR if $F_0 \leq F_0^*$; otherwise the firm chooses Procedure 1 with no ATR. Formally, the critical fee level F_0^* solves

$$EU^{(1)}(\theta^*) = EU^{(2)}(\theta^{**}) .$$

Applying Propositions 2 and 3 and $F_0 = F/(1 + r \cdot T)$, we obtain

$$F_0^* = \frac{1}{\gamma(1 + r \cdot T)} \ln \frac{EU^{(1)}(\theta^*)}{h(\tau, \eta_p, \eta_l)} , \tag{14}$$

with $h(\tau, \eta_p, \eta_l) := -\frac{1}{\gamma}\Big[\exp\{\gamma(1 - \tau\eta_p)X_0\} \cdot g_1(X_0, \tau\eta_p) + \exp\{\gamma(1 - \lambda\tau\eta_l)X_0\} \cdot g_2(X_0, \lambda\tau\eta_l)\Big]$.

So far, we have taken the viewpoint of the firm. The firm chooses the ATR (Procedure 2) if the offered fee F_0 is less than the critical fee F_0^*. Thus, F_0^* can be interpreted as the firm's maximal willingness to pay for the service provided by the ATR.

Tax authority's point of view

To compare the expected terminal tax revenue[9] under both tax procedures, we assume that an ATR is offered only if the expected terminal tax and fee revenue from this firm is at least as high as in the case without an ATR (the tax authority's procedural indifference). The tax authority is risk neutral with respect to tax uncertainty. Further, we assume that the tax authority cannot observe the risk aversion coefficient of the firm. Nor does it know the firm's specific investment strategies. However, the tax authority does know the distribution of the firm's terminal wealth at time T, i.e., X_T, based on which the tax authority can compute the expected terminal tax revenue. For each distribution of X_T, the tax authority can compute a critical fee level. In this context, the tax authority's aim is to determine a fee level that is at least revenue neutral. The critical fee level F_0^{**} is the lowest level the tax authority is willing to accept when offering an ATR for a given X_T distribution. At this level, the expected net revenue of the tax authority is zero, as additional fee revenues from ATR compensate exactly for the loss of tax revenue, due to lower investments and profits of taxpayers in response to the fee. As a consequence, it is reasonable for the tax authority to provide the ATR for a given X_T distribution, if the fee is higher than or equal to this critical level; i.e., $F_0 \geq F_0^{**}$.

Let us now derive the critical fee level of the tax authority F_0^{**}. As, in our setting, X_T is coupled with the investment amount, we obtain the expected terminal wealth of the authority as a function

[9] We assume that the tax authority maximizes revenue. This assumption is consistent with Graetz et al. (1986), Reinganum and Wilde (1986), Beck and Jung (1989), Sansing (1993); Elitzur and Mintz (1996), Mills and Sansing (2000), Mansori and Weichenrieder (2001), De Waegenaere et al. (2006); De Waegenaere et al. (2007), and Diller et al. (2025). Thus, the tax authority will not offer an ATR that does not increase tax revenue.

of θ straightforwardly. For Procedure 1 (no ATR), the expected terminal tax revenue from this firm is

$$
\begin{aligned}
ER^{(1)}(\theta) &:= \tau \cdot (d \cdot \delta_p + (1-d)) \cdot \mathbb{E}\big[(X_T - X_0)\mathbb{1}_{\{X_T \geq X_0\}}\big] \\
&\qquad - \lambda\tau \cdot (d \cdot \delta_l + (1-d)) \cdot \mathbb{E}\big[(X_0 - X_T)\mathbb{1}_{\{X_T < X_0\}}\big] \qquad (15) \\
&= \tau \cdot (d \cdot \delta_p + (1-d)) \cdot \Bigg[\Big(1 - \Phi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big)\Big)\big(\mathbb{E}[X_T] - X_0\big) \\
&\qquad + \varphi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big) \cdot \sqrt{\mathrm{Var}[X_T]}\Bigg] - \lambda\tau \cdot (d \cdot \delta_l + (1-d)) \\
&\qquad \cdot \Bigg[\Phi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big) \cdot \big(X_0 - \mathbb{E}[X_T]\big) + \varphi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big) \cdot \sqrt{\mathrm{Var}[X_T]}\Bigg],
\end{aligned}
$$

where the standard normal density and cumulative distribution function are given by $\varphi(t) := \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$ and $\Phi(x) := \int_{-\infty}^{x} \varphi(t)\mathrm{d}t$. In the last step, we use that the positive (respectively negative) part of $(X_T - X_0)$ follows a truncated normal distribution. Similarly, for Procedure 2 (ATR), the expected terminal tax revenue is given by

$$
\begin{aligned}
ER^{(2)}(\theta) &= \eta_p\tau \cdot \mathbb{E}\big[(X_T - X_0)\mathbb{1}_{\{X_T \geq X_0\}}\big] - \lambda\eta_l\tau \cdot \mathbb{E}\big[(X_0 - X_T)\mathbb{1}_{\{X_T < X_0\}}\big] + F \\
&= \tau \cdot \Bigg[\eta_p\big(\mathbb{E}[X_T] - X_0\big) - \big(\eta_p - \lambda\eta_l\big) \cdot \Phi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big)\Big) \cdot \big(\mathbb{E}[X_T] - X_0\big) \\
&\qquad + \big(\eta_p - \lambda\eta_l\big) \cdot \varphi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big) \cdot \sqrt{\mathrm{Var}[X_T]}\Bigg] + F\,. \qquad (16)
\end{aligned}
$$

As we have learned from Propositions 2 and 3, the firm invests different amounts in the risky project under the two tax procedures. The tax authority anticipates this behavior. Taking this into consideration, the critical fee level F_0^{**} is determined such that

$$ER^{(1)}(\theta^*) = ER^{(2)}(\theta^{**})\,. \qquad (17)$$

We therefore compute the fee level F_0^{**} satisfying equation (17). The parameters, such as δ_p, δ_l, η_p and η_l, are important drivers of the magnitude of F_0^{**}.

Overall, taking both the viewpoint of the firm and tax authority, an ATR is offered for an ATR fee $F_0 \geq F_0^{**}$. The firm accepts this fee according to its risk aversion and optimal investment amount if and only if this fee is lower than F_0^*. If $F_0^* < F_0^{**}$, an ATR will not be requested by the firm. Thus ATR fees with $F_0 \in [F_0^{**}, F_0^*]$, for $F_0^* \geq F_0^{**}$, are Pareto-improving to both the firm and the tax authority.

We have identified specific circumstances under which both the firm and the tax authority are more likely to benefit and thus $F_0^{**} < F_0^*$. Although the tax authority's revenue loss from offering an ATR remains limited (e.g., F_0^{**} remains limited), it becomes more likely that $F_0^{**} < F_0^*$, when the firm's potential losses from tax uncertainty are especially severe—so that its willingness to pay for certainty is high. In particular, if the parameters that drive tax uncertainty (e.g., a high audit probability d combined with a high profit-side risk multiplier δ_p and/or a low loss offset parameter λ with a low loss-side multiplier δ_l) are such that the downside risk for the firm is large. Then the firm's maximal willingness to pay F_0^* increases. At the same time, if the tax authority can set the ATR's deterministic rates (through η_p and η_l) close to the original expected tax rates—thereby keeping the reduction in tax revenue small—then the authority's minimum acceptable fee F_0^{**} will be relatively low. Under these circumstances, the gap widens so that $F_0^{**} < F_0^*$, making it feasible for both parties to agree on an ATR fee that is acceptable to the tax authority and still attractive enough for the firm to request the ATR.

3 Numerical analyses

To gain an intuitive understanding of the results stated in Section 2, Propositions 2 and 3, and the discussions about the critical fees, we conduct numerical analyses and therefore consider some parameters as given. In our baseline scenario, we assume

$$\mu = 0.06, \quad r = 0.02, \quad \sigma = 0.15, \quad \gamma = 0.02, \quad T = 5, \quad X_0 = 100, \quad \tau = 15\%\,.$$

The rate of return μ on the risky project and the volatility σ are chosen in line with return and volatility of companies listed in the MSCI World Index. At the end of Section 3.1, we vary both parameters starting from this baseline scenario. The parameter choice leads to a Sharpe ratio of $26.67\% \approx (\mu - r)/\sigma$. The Sharpe ratio describes the performance of a risky project in relation to a risk-free investment. Typically, a Sharpe ratio above 0.5 in the long run indicates great investment performance and is difficult to achieve, while a ratio between 0.1 and 0.3 is frequently considered reasonable and can be achieved more easily (e.g., Sharpe 1994). We base our numerical analysis of the optimal investment amount on a 15% tax rate, reflecting the global minimum tax introduced under the OECD's Pillar Two framework. This initiative, endorsed by more than 140 countries, seeks to ensure that multinationals pay an effective corporate tax rate of at least 15% on their worldwide profits, thereby curbing harmful tax competition and profit shifting to low-tax jurisdictions. The choice of the risk aversion parameter γ is based on experimental studies.[10]

[10]Experimental studies find reasonable parameters for the relative risk aversion (RRA) coefficient (e.g., Barsky et al. 1997). For the underlying exponential utility function, the absolute relative risk aversion (ARA) is constant and denoted by γ, while the RRA for exponential utility varies in time. For our numerical example, we deduce the ARA coefficient, i.e., γ, of the exponential utility from the observed RRA level in the literature by assuming that $\gamma \cdot X_0$ = observed RRA levels. Barsky et al. (1997) show, for 65 percent of the data, an RRA above 3.76, 24 percent below 2, and 12 percent between 2 and 3.76. Thus, we use $\gamma = 0.02$ and use $\gamma = 0.03$ in the following.

We relax these assumptions in subsequent numerical analyses to demonstrate the robustness and sensitivities of our baseline scenario.

3.1 Optimal investment amount

Let us now analyze the optimal investment amount and the implications of both tax procedures, choosing the parameters set in our baseline scenario and six different choices of the parameters ($\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l,\ F_0,\ \tau$). To illustrate the effect of tax uncertainty on the optimal investment amount in the case of R&D investments, in Table 1, we choose a rather high tax uncertainty scenario with an audit probability varying from $d = 40\%$ to 60%. In the literature, audit probabilities vary considerably within and across countries (Mendoza et al. 2017; Bachas et al. 2019). There is evidence for extremely low audit probabilities with realistic magnitudes, ranging from 1 percent to 3 percent (Dhami and al Nowaihi 2007), as well as high audit rates, ranging from 20 percent to 50 percent (e.g., Collins and Plumlee 1991; Alm et al. 1992; Alm et al. 1993; Andreoni et al. 1998; Bernasconi 1998). In Hoopes et al. (2012), the average estimated IRS audit rate for U.S. public firms between 1992 and 2008 is 29 percent, ranging from 0 percent to 55 percent with 19 percent in the 25th percentile to 37 percent in the 75th percentile of their data. Given that R&D is a matter with high-tax uncertainty, due to the inherently hard-to-value intangibles and transfer pricing issues, assuming even higher audit probabilities seems appropriate.

If there is an audit, the profit tax collected is assumed to be $\delta_p = 4$ (scenario (a)$-$(c)) and $\delta_p = 3$ (scenario (d)$-$(f)) times higher than originally declared.[11] To allow for a better interpretation of the resulting numbers, we display the certainty equivalent (CE) instead of the optimal utility level. The use of the CE makes the quantities easier to interpret, because the CE expresses the expected utility in monetary units instead of utility units. In our case, we can define the certainty equivalent as an amount of certain capital that the firm shall receive as an equivalent for the uncertain terminal wealth arising from the R&D investment. As the argument of the utility is $X_T - X_0$, we define the certainty equivalent as follows

$$CE^{(1)} = -\frac{1}{\gamma}\ln\left(-\gamma EU^{(1)}(\theta^*)\right) + X_0\,,\quad CE^{(2)} = -\frac{1}{\gamma}\ln\left(-\gamma EU^{(2)}(\theta^{**})\right) + X_0\,,$$

where $CE^{(1)}$ and $CE^{(2)}$ give the certainty equivalents resulting from the case without and with an ATR (Procedures 1 and 2), respectively. Table 1 illustrates the optimal investment amount and the certainty equivalent of the firm under the two tax procedures for various parameter combinations

[11] These assumptions account for the pronounced exposure of R&D investments to tax uncertainty. Disputes on what is tax deductible and transfer pricing may easily lead to a multiple times higher tax burden following a tax audit (e.g., 9 billion dollars of potential additional taxes for Facebook, White (2020) and 9 billion dollars of expected additional taxes from a transfer pricing dispute of Coca-Cola, Byrnes (2024). We conducted sensitivity analyses using, for example, $\delta_p = 1.5$ and found the same qualitative effects as discussed in the following. The general effect of tax uncertainty on optimal investment is, however, diminishing.

of (δ_p, δ_l, d, η_p, η_l, F_0, τ) and two choices of risk aversion γ. We have chosen an ATR fee F_0 inside the acceptance set $[F_0^{**}, F_0^{*}]$.[12]

Table 1 Optimal investment amount and certainty equivalents for Procedures 1 and 2 for relatively generous loss offset and different levels of risk aversion

Panel A: Optimal investments with a level of risk aversion of $\gamma = 0.03$					
	δ_p δ_l d η_p η_l F_0 τ	θ^*	$CE^{(1)}$, no ATR	θ^{**}	$CE^{(2)}$, ATR
#		$\lambda = 0.9$, $\gamma = 0.03$			
(a)	$(4.00, 1.00, 0.40, 2.20, 1.00, 0.00, 0.15)$	62.88	111.04	65.52	111.48
(b)	$(4.00, 1.00, 0.50, 2.50, 1.00, 0.00, 0.15)$	62.03	110.26	64.97	110.71
(c)	$(4.00, 1.00, 0.60, 2.80, 1.00, 0.00, 0.15)$	61.12	109.51	64.15	109.93
(d)	$(3.00, 1.00, 0.40, 1.80, 1.00, 0.00, 0.15)$	65.00	112.28	65.93	112.46
(e)	$(3.00, 1.00, 0.50, 2.00, 1.00, 0.00, 0.15)$	64.76	111.78	65.77	111.97
(f)	$(3.00, 1.00, 0.60, 2.20, 1.00, 0.00, 0.15)$	64.51	111.29	65.52	111.48

Panel B: Optimal investments with a lower level of risk aversion of $\gamma = 0.02$					
	δ_p δ_l d η_p η_l F_0 τ	θ^*	$CE^{(1)}$, no ATR	θ^{**}	$CE^{(2)}$, ATR
#		$\lambda = 0.9$, $\gamma = 0.02$			
(a)	$(4.00, 1.00, 0.40, 2.20, 1.00, 0.00, 0.15)$	93.51	113.03	97.33	113.55
(b)	$(4.00, 1.00, 0.50, 2.50, 1.00, 0.00, 0.15)$	91.60	112.04	95.87	112.57
(c)	$(4.00, 1.00, 0.60, 2.80, 1.00, 0.00, 0.15)$	89.55	111.07	93.96	111.58
(d)	$(3.00, 1.00, 0.40, 1.80, 1.00, 0.00, 0.15)$	97.35	114.60	98.71	114.82
(e)	$(3.00, 1.00, 0.50, 2.00, 1.00, 0.00, 0.15)$	96.61	113.96	98.09	114.19
(f)	$(3.00, 1.00, 0.60, 2.20, 1.00, 0.00, 0.15)$	95.84	113.33	97.33	113.55

Notes: This table displays the optimal investment amounts and certainty equivalents under Procedure 1 (no ATR) and Procedure 2 (ATR) for different parameter sets for either a higher (Panel A) or lower level of risk aversion (Panel B). We assume different sets of parameters for our numerical analysis as displayed in rows (a)-(f), including those from our baseline scenario, i.e., a rate of return on the risky investments of $\mu = 0.06$, risk-free rate $r = 0.02$, volatility $\sigma = 0.15$, an investment horizon $T = 5$, initial wealth $X_0 = 100$, tax rate $\tau = 0.15$, a tax loss offset parameter $\lambda = 0.9$ for risk aversion at the level of $\gamma = 0.03$ (Panel A) or a lower level of risk aversion of $\gamma = 0.02$ (Panel B). The profit and loss tax risk multipliers δ_p, δ_l, the probability of a tax reduction after a post-tax audit d are only relevant for the case without ATR (columns θ^* and $CE^{(1)}$), while the profit and loss tax multipliers under ATR-induced tax certainty η_p, η_l and the ATR fee F_0 are exclusively relevant for the case with ATR (columns θ^{**} and $CE^{(2)}$).

We highlight three key insights that emerge from comparing the two scenarios—with and without an ATR. First, recall that, with an ATR, the random tax rate $\tilde{\tau}_p$ ($\tilde{\tau}_l$) is replaced by the fixed tax rate $\eta_p \cdot \tau$ ($\eta_l \cdot \tau$), provided the firm pays the ATR fee F_0. Across all scenarios (a)–(f) and for both levels of risk aversion, we consistently find that the optimal investment level under tax uncertainty (θ^*) is strictly lower than the optimal investment level when tax certainty is secured through an ATR (θ^{**}). This finding is intuitive, as resolving tax uncertainty enables the firm to pursue riskier investments and, in turn, to tolerate greater cash flow uncertainty.

Second, we find that more generous loss offset provisions fuel risk-taking. To illustrate this amplifying effect of loss offsets and ATRs, in Table 2, we vary the loss offset parameter λ, keeping the other parameters as in Table 1 and focusing on a uniform risk aversion level of $\gamma = 0.02$. If the firm has no possibility to offset losses ($\lambda = 0$), it will invest less in the risky project. In comparison, the risky investment amount θ^* is comparably larger, if the loss offset parameter rises

[12] We provide a detailed discussions of reasonable ATR fees at the end of this section.

to 0.9 (c.f. Table 1) or if even a full offset ($\lambda = 1$) is possible. This finding is consistent with Ljungqvist et al. (2017), Langenmayr and Lester (2018), and Osswald and Sureth-Sloane (2024). Here, a tight tax loss offset provision "punishes" losses, as they are not fully deductible. Then more conservative (less risky) investments with a lower probability of losses (lower θ) become more attractive. In addition, the tax offset parameter λ also has a monotone effect on the optimal investment amount θ^{**} under Procedure 2 (ATR). Compared to the case with no ATR, this effect is more pronounced. Specifically, for any given tax offset parameter λ, the application of ATR leads to a riskier investment; i.e., $\theta^{**} > \theta^{*}$. Implementing the ATR together with a generous tax loss offset policy will further encourage risk-taking.

Third, under both tax procedures, a more risk-averse firm (higher risk aversion parameter $\gamma = 0.03$) will invest less in the risky project than a less risk-averse firm ($\gamma = 0.02$). Moving from $\gamma = 0.02$ to 0.03, the firm reduces its holdings in the risky project. The magnitude of this decrease in the optimal investment amount does not differ substantially between the two tax procedures and is slightly higher in case of an ATR (i.e., θ^{**}) than in the case without ATR (i.e., θ^{*}). Surprisingly, an ATR has a larger impact for a less risk-averse firm. Through the "guaranteed" deterministic tax rates, the less risk-averse firm will undertake riskier investments, expecting to be rewarded with a higher expected return. If the firm is rather risk-averse, the guaranteed tax rates become less interesting to the firm, as it will per se pursue a less risky alternative investment. In this sense, the reduction of the optimal amount invested in a risky project caused by the increase in risk aversion is less substantial for the case with no ATR than for the case with an ATR.

To examine how tax (uncertainty) impacts optimal investment and cashflow uncertainty, we build on Proposition 1. In Proposition 1, equation (9), we have a simple relation between a constant tax rate τ and the optimal investment amount, namely:

$$\theta^s = \min\left\{\frac{\mu - r}{\gamma\sigma^2(1-\tau)},\, X_0\right\}.$$

The optimal investment amount is a function of the adjusted Sharpe ratio SR$= (\mu - r)/\sigma^2$. As Propositions 2 and 3 demonstrate, this relation gets more complicated and difficult to grasp under asymmetric taxation of profits and losses and even more challenging in the case of tax uncertainty. Figure 1 indicates that, in Procedure 2 with an ATR, a simple relation as equation (9) does not hold.

To illustrate this, we compare investments with different volatility σ (x-axis). To obtain comparables, the rate of return μ of these investments is adapted such that the Sharpe ratio is the same for each of the underlying investments considered.[13] Using the parameter set of scenario (a) from

[13]For the analysis, we look at different investment opportunities with identical adjusted Sharpe ratios; i.e., for different volatility σ, we choose pairs $(\mu, \sigma^2) = \left(\mathrm{SR}\cdot\sigma^2 + r, \sigma^2\right)$ and fix the adjusted Sharpe ratio SR ≈ 1.77778 using the base case parameter set.

Table 2 Optimal investment amount and certainty equivalents for Procedures 1 and 2 under different loss offset provisions (full or no loss offset)

Panel A: Optimal investments without tax loss offset					
	δ_p δ_l d η_p η_l F_0 τ	θ^*	$CE^{(1)}$	θ^{**}	$CE^{(2)}$
#		$\lambda = 0$, $\gamma = 0.02$			
(a)	$(\mathit{4.00}, 1.00, \mathit{0.40}, \mathit{2.20}, 1.00, 0.00, 0.15)$	78.60	112.17	81.84	112.61
(b)	$(\mathit{4.00}, 1.00, \mathit{0.50}, \mathit{2.50}, 1.00, 0.00, 0.15)$	76.59	111.24	80.20	111.68
(c)	$(\mathit{4.00}, 1.00, \mathit{0.60}, \mathit{2.80}, 1.00, 0.00, 0.15)$	74.45	110.33	78.15	110.74
(d)	$(\mathit{3.00}, 1.00, \mathit{0.40}, \mathit{1.80}, 1.00, 0.00, 0.15)$	82.31	113.63	83.50	113.82
(e)	$(\mathit{3.00}, 1.00, \mathit{0.50}, \mathit{2.00}, 1.00, 0.00, 0.15)$	81.45	113.02	82.74	113.22
(f)	$(\mathit{3.00}, 1.00, \mathit{0.60}, \mathit{2.20}, 1.00, 0.00, 0.15)$	80.54	112.42	81.84	112.61

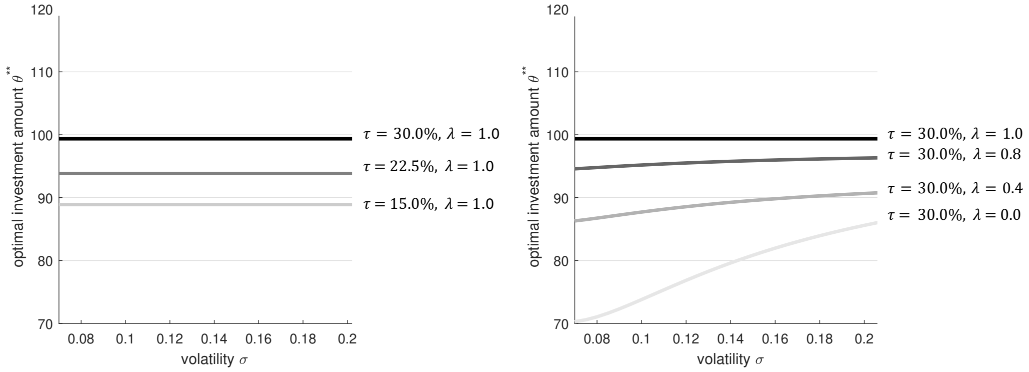
Panel B: Optimal investments with a full tax loss offset					
	δ_p δ_l d η_p η_l F_0 τ	θ^*	$CE^{(1)}$	θ^{**}	$CE^{(2)}$
#		$\lambda = 1$, $\gamma = 0.02$			
(a)	$(\mathit{4.00}, 1.00, \mathit{0.40}, \mathit{2.20}, 1.00, 0.00, 0.15)$	95.46	113.14	99.34	113.68
(b)	$(\mathit{4.00}, 1.00, \mathit{0.50}, \mathit{2.50}, 1.00, 0.00, 0.15)$	93.57	112.14	97.92	112.69
(c)	$(\mathit{4.00}, 1.00, \mathit{0.60}, \mathit{2.80}, 1.00, 0.00, 0.15)$	91.54	111.17	96.04	111.69
(d)	$(\mathit{3.00}, 1.00, \mathit{0.40}, \mathit{1.80}, 1.00, 0.00, 0.15)$	99.30	114.73	100.00	114.95
(e)	$(\mathit{3.00}, 1.00, \mathit{0.50}, \mathit{2.00}, 1.00, 0.00, 0.15)$	98.58	114.08	100.00	114.32
(f)	$(\mathit{3.00}, 1.00, \mathit{0.60}, \mathit{2.20}, 1.00, 0.00, 0.15)$	97.83	113.45	99.34	113.68

Notes: This tables displays the optimal investment amounts and certainty equivalents under Procedure 1 (no ATR) and Procedure 2 (ATR) for different parameter sets and either without (Panel A) or without (Panel B) tax loss offset. We assume different sets of parameters for our numerical analysis, as displayed in rows (a)-(f), including those from our baseline scenario, i.e., a rate of return on the risky investments of $\mu = 0.06$, risk-free rate $r = 0.02$, volatility $\sigma = 0.15$, an investment horizon $T = 5$, initial wealth $X_0 = 100$, tax rate $\tau = 0.15$, risk aversion at the level of $\gamma = 0.02$ for either no tax loss offset $\lambda = 0$ (Panel A) or full tax loss offset $\lambda = 1$ (Panel B). The profit and loss tax risk multipliers δ_p, δ_l, the probability of a tax reduction after a post-tax audit d are only relevant for the case without ATR (columns θ^* and $CE^{(1)}$) while the profit and loss tax multipliers under ATR-induced tax certainty η_p and η_l and the ATR fee F_0 are exclusively relevant for the case with ATR (columns θ^{**} and $CE^{(2)}$).

Table 1 Panel B, the left-hand side confirms equation (9) for the case of symmetric taxation ($\lambda = 1$, full loss-offset) and different tax rates τ. Being a constant function, the optimal investment amount θ^{**} only depends on the adjusted Sharpe ratio and not directly on the volatility σ. The right-hand side provides the same graph for different choices of $\lambda < 1$. Clearly, the optimal investment amount now varies with the volatility σ of the investment and is not just a function of the adjusted Sharpe ratio $\mathrm{SR} = (\mu - r)/\sigma^2$. The dashed line highlights the case of $\sigma = 15\%$ from Table 1.

To the best of our knowledge, this is the first study on ATRs to examine simultaneously cash-flow and tax uncertainty. If taxes are constant or cash flow deterministic, this immediately implies that cash flow and taxes are independent from each other. In Diller et al. (2017), who abstract from pre-tax cash-flow uncertainty and its interaction with tax uncertainty, the willingness to pay for an ATR is linked to the variance of the after-tax net payoff $\tilde{X}_T^{(1)}$. As investments are deterministic, the highest variance is (in all cases) reached for the highest tax uncertainty. Mathematically, in their model, the variance of the profit tax is explicitly given by $\mathrm{Var}(\tilde{\tau}_p) = \tau^2(1-\delta_p)^2 d(1-d)$, where d is the probability of a tax audit. Obviously, this is maximized by $d = 0.5$; see also the left-hand side

Fig. 1 Optimal investment amount θ^{**} (Procedure 2, ATR) for different tax rates and loss offset provisions



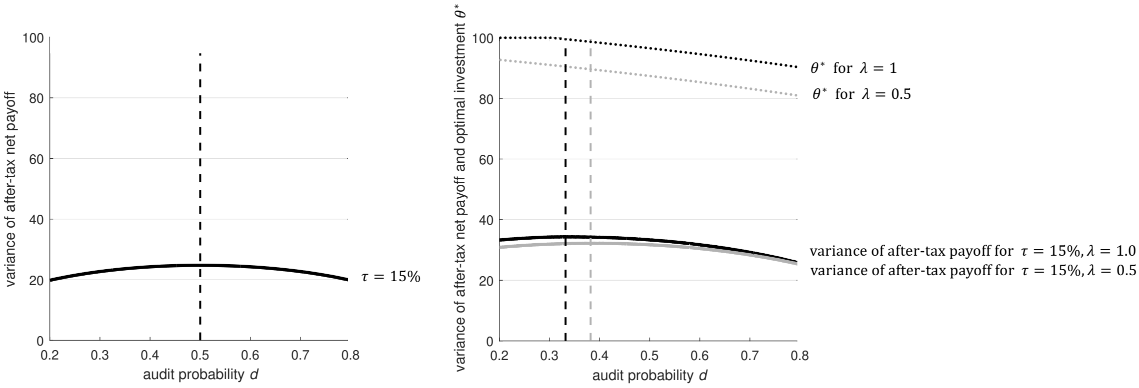
Notes: This figure illustrates the optimal investment amount under Procedure 2 (ATR) as a function of the investment volatility for different tax rates or loss offset restrictions. We choose scenario (a) from Table 1, Panel B, i.e. $r = 0.02$, $T = 5$, $X_0 = 100$, $\tau = 15\%$, $\delta_p = 4$, $\delta_l = 1$, $d = 0.4$, $F_0 = 0$, $\gamma = 0.02$. The left graph exemplifies symmetric taxation, i.e., taxation with full loss offset ($\eta_p = \eta_l = 1$, $\lambda = 1$). Under full loss offset, the optimal investment amount is insensitive to σ. By contrast, the right graph exemplifies asymmetric taxation, i.e., loss offset restrictions of different levels ($0 \leq \lambda \leq 1$). We choose the rate of return on risky investments/risk pairs (μ, σ^2) with identical adjusted Sharpe ratio SR = 1.77778.

of Figure 2. By contrast, in our framework, the optimal investment amount is a response to tax uncertainty. This means that a higher tax uncertainty is usually compensated for by a reduction in the optimal investment amount. Thus, the highest variance of the after-tax net payoff does no longer coincide with the highest variance of the tax rate, i.e., the highest tax risk. It is no longer obvious which audit probability leads to the highest variance of the after-tax net payoff $\tilde{X}_T^{(1)}$. Given parameters from scenario (1) from Table 1, we find that this optimal audit probability can now also take values between 0.3 and 0.4 and depends, for example, on the loss-offset parameter λ; see the right-hand side of Figure 2. For tax authorities, this identifies situations where it is most beneficial to introduce an ATR.

3.2 Pareto improving ATR fees

To illustrate our theoretical findings from Section 2 on the choice of Pareto improving ATR fees, we investigate when $F_0^{**} < F_0^*$ and hence under what conditions the tax authority and the firm agree on an ATR contract. Under the given set of assumptions, Table 3 provides the viewpoint of firms for various risk aversion levels γ and displays the resulting critical fee levels F_0^*. Recall that F_0^* is the maximum fee the firm is willing to pay for the ATR. Starting with a specific risk aversion level (in our example, $\gamma = 0.04$), we observe that a more risk averse firm (with a higher risk aversion level) is willing to pay more to eliminate tax uncertainty by requesting the ATR. However, the impact of risk aversion on the maximal willingness to pay F_0^* is not obvious, as can be seen in Table 3. We

Fig. 2 Variance of the after-tax net payoff (Procedure 1, no ATR) and optimal investment amount θ^* as a function of audit probability d for scenario (a) of Table 1.



Notes: This figure illustrates the variance of the after-tax net payoff as a function of the audit probability d (left-hand side) and the variance of the after-tax net payoff or optimal investment amount as a function of the audit probability d both under Procedure 1 (no ATR). We choose scenario (a) from Table 1 Panel B, i.e. $r = 0.02$, $T = 5$, $X_0 = 100$, $\tau = 15\%$, $\delta_p = 4$, $\delta_l = 1$, $d = 0.4$, $F_0 = 0$, $\gamma = 0.02$. The left graph shows a risk-free investment ($\theta = 0$). The right graph displays the optimal investment amount θ^* and the variance of the after-tax net payoff. The vertical line describes the highest variance of the after-tax net payoff $\tilde{X}_T^{(1)}$.

have to implicitly determine the critical fee F_0^* by equating the optimal expected utility for the tax procedure with no ATR and the procedure with a fee-based ATR. As a higher risk aversion affects the expected utility for both procedures, its impact on the maximal willingness to pay F_0^* is not monotone and may lead to either an increase or decrease. This finding can be confirmed by the numerical results for F_0^* in Table 3, which also are not monotone in the risk aversion level γ.

Table 3 Critical fee levels F_0^* for different levels of risk aversion

risk aversion	$\gamma = 0.02$	$\gamma = 0.03$	$\gamma = 0.04$	$\gamma = 0.05$	$\gamma = 0.06$	$\gamma = 0.07$	$\gamma = 0.08$
F_0^*	0.132	0.118	0.107	0.102	0.104	0.112	0.114

Notes: This table displays critical fee levels for different levels of risk aversion. We use the set of parameters of the baseline scenario, i.e., a rate of return on the risky investments of $\mu = 0.06$, risk-free rate $r = 0.02$, volatility $\sigma = 0.15$, an investment horizon $T = 5$, initial wealth $X_0 = 100$, tax rate $\tau = 0.15$, for various levels of risk aversion γ. We assume profit and loss tax risk multipliers $\delta_p = 3.0$ and $\delta_l = 0.5$ and a probability of a tax reduction after a post-tax audit of $d = 0.2$ and a tax loss offset parameter $\lambda = 0.5$. We assume profit and loss tax multipliers under ATR-induced tax certainty $\eta_p = 1.40$ and $\eta_l = 0.90$.

In our example, we observe a U-shaped relationship between risk aversion γ and the critical fee level F_0^*, reflecting how firms balance the trade-off between tax uncertainty and the cost of achieving certainty through an ATR. Here, F_0^* represents the firm's willingness to pay the ATR fee to ensure a fixed tax rate, as opposed to facing the risk of a change in the applied tax rate after a tax audit, which could result in higher taxes. The U-shaped effect results from two competing effects.

First, because firms dislike any kind of uncertainty, those with a higher risk aversion are generally willing to pay more for (investment or tax) uncertainty. As a result, their willingness to pay to secure a fixed tax rate remains relatively high. These firms prefer safe, low-risk investments, and their overall strategy revolves around avoiding volatility and exposure to potential negative outcomes. As such, the possibility of facing a random, potentially higher tax rate after an audit is particularly unappealing. To avoid this uncertainty, they are willing to pay a higher fee for the ATR, which ensures a fixed and predictable tax rate. This suggests that the critical fee level F_0^* increases with risk aversion.

Second, firms with lower risk aversion optimally invest more in the risky project (higher θ^* and θ^{**}). The increased risk comes with a higher expected return and thus also a higher expected tax base. While for $\gamma = 0.02$ the optimal investment amount is $\theta^* \approx 99$, it drops to $\theta^* \approx 26$ for $\gamma = 0.08$. This implies that tax risk is much more relevant for a firm with low risk aversion, increasing its willingness to pay for an ATR. Due to the second effect, we observe a high willingness to pay for an ATR for firms with low risk aversion. The willingness to pay drops for medium risk aversion and, due to the first effect, increases again for higher risk aversions, leading to the U-shape. This U-shape was also observed for many different parameter choices and persists for tax-loss-offset parameters $\lambda \in [0, 1]$.

To some extent, the firm can compensate for a higher tax risk with a lower investment risk. We demonstrate this by noting that the optimal investment amount θ^* is lower than θ^{**}. This helps mitigate the increase of the critical fee level F_0^* for low risk aversion but does not affect the U-shape.

In summary, the U-shaped relationship between risk aversion and the critical fee reflects the varying degrees of sensitivity to tax uncertainty across different levels of risk aversion. Firms with low risk aversion are willing to pay a higher fee to avoid the significant downside risks associated with audits and variable taxation, despite their general comfort with risk. Firms with moderate risk aversion are more balanced in their approach, leading to a lower willingness to pay for tax certainty and thus a lower critical fee. Highly risk-averse firms, in contrast, are most concerned with eliminating uncertainty, and their willingness to pay a higher fee for an ATR reflects their desire to secure a fixed tax rate, despite the generally lower returns from their conservative investments. This U-shaped pattern illustrates how firms' tax strategies and willingness to pay for certainty evolve with their risk preferences.

Table 4 takes the view of tax authorities and exhibits the critical fee level F_0^{**} for various combinations of tax rate multipliers η_p, η_l of the ATR. According to our considerations in Section 2, the tax authority is willing to offer an ATR for any fee higher than or equal to F_0^{**}. Recall that $\eta_p \cdot \tau$ and $\eta_l \cdot \tau$ are the ultimately applied tax rates on profits and losses, respectively, as agreed on under the ATR. Both an increase in η_p and a decrease in η_l lead to higher average tax revenue for the tax authority. A higher tax on profits and a lower tax on losses both increase the overall

Table 4 Critical fee level F_0^{**} for different tax rate multipliers

tax rate multiplier		F_0^{**}
$\eta_p = 1.30$,	$\eta_l = 1.00$	0.25
$\eta_p = 1.35$,	$\eta_l = 0.95$	0.12
$\eta_p = 1.40$,	$\eta_l = 0.90$	−0.02
$\eta_p = 1.45$,	$\eta_l = 0.85$	−0.15
$\eta_p = 1.50$,	$\eta_l = 0.80$	−0.28
$\eta_p = 1.55$,	$\eta_l = 0.75$	−0.42
$\eta_p = 1.60$,	$\eta_l = 0.70$	−0.55
$\eta_p = 1.65$,	$\eta_l = 0.65$	−0.68
$\eta_p = 1.70$,	$\eta_l = 0.60$	−0.81

Notes: This table displays critical fee levels for different tax rate multipliers. We use the set of parameters of the baseline scenario, i.e., a rate of return on the risky investment of $\mu = 0.06$, risk-free rate $r = 0.02$, volatility $\sigma = 0.15$, an investment horizon $T = 5$, initial wealth $X_0 = 100$, tax rate $\tau = 0.15$, a tax loss offset parameter $\lambda = 0.5$ and a level of risk aversion $\gamma = 0.04$. We assume profit and loss tax risk multipliers $\delta_p = 3.0$ and $\delta_l = 0.5$ and a probability of a tax reduction after a post-tax audit of $d = 0.2$ and a loss offset parameter $\lambda = 0.5$. We provide critical fee levels F_0^{**} for various sets of profit and loss tax multipliers under ATR-induced tax certainty η_p and η_l.

tax payment. Consistently, Table 4 illustrates that the resulting critical fee F_0^{**} the tax authority requires decreases with η_p and $-\eta_l$.

The implementation of the ATR in case of a relatively low ATR profit tax η_p and a relatively high η_l requires a positive fee for the ATR. On the contrary, if the tax authority has already implemented a relatively high ATR profit tax (i.e., high η_p) and a relatively strict loss offset provision (i.e., low η_l), the application of the ATR can be realized by a zero or negative fee. A negative F_0^{**} implies that any positive fee is acceptable for the tax authority and the authority is even willing to pay the firm (negative fee) if the firm requests the offered ATR. Such a negative fee can be interpreted as the tax authority's willingness to invest in the ATR, for example, in human resources or technology, and thereby reduce taxpayers' compliance costs and make the ATR more attractive. Our results clarify that such a strategy benefits the tax authority for several sets of (fixed) tax rates. Note that, for Table 3, we assumed $\eta_p = 1.40$, $\eta_l = 0.90$. Under this set of assumptions, the firm's resulting maximal willingness to pay for the ATR is positive for all different risk aversion levels. For this combination of η_p and η_l, the minimum fee required by the tax authority is negative. As a consequence, both the firm and the tax authority are ready to enter into the ATR. In this scenario, firms and the authority will always agree on the ATR for fees within the interval $[F_0^{**}, F_0^{*}]$.

In our numerical illustrations, we confirm and quantify the theoretical findings in section 2. We find that, if a firm chooses an ATR, the ATR allows it to make riskier investments. More importantly, the tax authority can offer an acceptable ATR via different combinations of ATR fees on the one side and tax rates for profits and losses on the other. For example, the tax authority can choose a low profit tax rate and a positive ATR fee or a high profit tax and a negative ATR fee to achieve at least expected revenue neutrality (i.e., zero expected revenue). This finding implies that, in high tax scenarios with considerable tax uncertainty, tax authorities should offer ATRs at low or even negative fees to stimulate risky investments.

4 Conclusion

We investigate the impact of an ATR on firm's risky investments and how ATR fees can be set to benefit both the firm and the tax authority. Contributing to the literature on taxation, advance tax rulings, and optimal asset allocation, we show that ATRs can help foster risky investments. We also determine the reasonable fee range for them. For most scenarios, a range can be derived, and hence an ATR can be contracted between the firm and the tax authority. Interestingly, our study demonstrates a U-shaped relationship between firms' risk aversion and their willingness to pay for tax certainty: both low and high levels of risk aversion drive an increase in the fee firms are willing to pay to mitigate potential audit-related tax risks. Further, under specific conditions, such as high ATR tax rates, a tax authority may be able to offer the ATR at a zero or even negative fee to maintain revenue neutrality.

We further show that the firm's and the tax authority's decisions to agree on a fee-based ATR crucially depend on the loss offset provision and profit tax rate as well as the firm's risk aversion level. A generous tax loss offset amplifies the ATR's potential to encourage risk taking. We find that the implementation of the ATR increases the riskiness of the projects, irrespective of the firm's risk aversion. However, this ATR-induced increase in riskiness is more pronounced for a less risk-averse firm because firms with low risk aversion value certainty to secure high expected returns, while highly risk-averse firms are especially sensitive to residual uncertainty and may be willing to pay a premium for certainty. By contrast, firms with moderate risk aversion exhibit the lowest willingness to pay for ATRs, as neither the investment incentive nor their aversion to uncertainty is sufficient to justify paying a high fee.

Our model is limited by its assumptions. The return from the risky investments is assumed to follow a normal distribution. In future research, more complex asset dynamics, moving beyond normally distributed returns and describing the evolution of the risky investments with a more general stochastic volatility model, in the sense of Heston (1993), could be developed. A stochastic volatility model would allow researchers to generalize our model and capture risk patterns that can be often observed in financial markets, such as heavy tails, volatility clustering, and the smile of implied volatilities (Tankov 2003). The incorporation of volatility risk intensifies the cash flow uncertainty. Hence, it would be interesting to investigate whether our findings on the impact of the ATR and loss offsets on asset allocation prevail in a more volatile market. Further, while we assumed exponential utility, i.e., a constant absolute risk aversion preference, to describe the firm's risk aversion, future research should consider behavioral risk and loss attitudes, as known, for example, from cumulative prospect theory. These extensions could help capture possible gambling by firms.

While the closed-form solutions derived here rely on the assumption of exponential utility, it is reasonable to believe that the main results will still hold under different utility functions, including preferences with nonmonotonic risk aversion, such as the SAHARA (Symmetric Asymptotic Hyperbolic Absolute Risk Aversion) utility (Chen et al. 2011). However, moving to such utility functions may introduce additional complexity and potentially necessitate more complex numerical or approximate solutions, as closed-form expressions may no longer be achievable. Future research could explore this direction further, quantifying the impact of different utility choices on the optimal allocation while considering the practical limitations of departing from closed-form results.

Our results demonstrate that the negative impact of tax uncertainty on risky investment can be substantially mitigated—or even reversed—by appropriately designed, low-cost ATR schemes. Specifically, our model predicts that countries with high tax rates are more successful in attracting risky investment if they offer low-cost ATRs in combination with generous loss offset provisions, compared to countries that impose high ATR fees. These findings underscore the importance for policymakers of carefully calibrating ATR fees and integrating them with effective loss offset rules. Importantly, our results suggest that, under certain conditions, tax authorities may need to subsidize ATRs—setting zero or even negative fees—to achieve investment and revenue goals, particularly in high-tax or high-uncertainty environments.

Our analysis also reveals that firms respond heterogeneously to ATR fees, with the lowest critical fee observed for firms with intermediate levels of risk aversion. This U-shaped relationship implies that both highly risk-averse and risk-tolerant firms are willing to pay more for tax certainty, while moderately risk-averse firms are less inclined to do so.

We encourage empirical research to test our predictions, for example, by exploiting variation in ATR fees across countries or by examining related tax certainty mechanisms, such as cooperative compliance programs or simplified audits. Such evidence would help tax authorities refine policies to reduce tax-induced investment frictions and foster risk-taking. Our findings also inform firms about the conditions under which applying for ATRs is most beneficial, supporting more informed asset allocation decisions that account for legislative, administrative, and judicial tax uncertainty.

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APPENDIX

A Variables

Table 5 Definition of variables

variable	definition
$CE^{(1)}$	certainty equivalent (no ATR)
$CE^{(2)}$	certainty equivalent (ATR)
d	probability of a tax reduction after a post-tax audit
$ER^{(1)}(\theta)$	expected terminal tax revenue (no ATR)
$ER^{(2)}(\theta)$	expected terminal tax revenue (ATR)
$EU^{(1)}(\theta)$	expected utility (no ATR)
$EU^{(2)}(\theta)$	expected utility (ATR)
$F = F_0(1 + r \cdot T)$	accumulated ATR fee
F_0	ATR fee
F_0^*	maximal willingness to pay for an ATR
F_0^{**}	minimal ATR fee
θ^*	optimal amount, asymmetric taxation, ATR
θ^{**}	optimal amount, asymmetric taxation, no ATR
θ^s	optimal amount $\theta^{**} = \theta^s$, symmetric taxation, ATR
r	risk-free rate
r_τ	interest rate after tax
R_T	time-T risky asset return
SR	Sharpe ratio
T	investment horizon
$U(X)$	utility function
X_0	initial wealth
X_T	terminal wealth of the firm at time T
$\tilde{X}_T^{(1)}$	after-tax net payoff (no ATR)
$\tilde{X}_T^{(2)}$	after-tax net payoff (ATR)
δ_l	tax risk multiplier for losses
δ_p	tax risk multiplier for profits
γ	risk aversion coefficient
η_l	tax rate multiplier for losses under ATR-induced tax certainty
η_p	tax rate multiplier for profits under ATR-induced tax certainty
θ	amount invested in the risky project
λ	tax loss offset parameter
μ	rate of return on risky investments
σ	volatility
τ	proportional tax rate for profits and losses
$\tilde{\tau}_l$	stochastic proportional tax rate for losses
$\tilde{\tau}_p$	stochastic proportional tax rate for profits
$\Phi(x)$	standard normal distribution function
$\varphi(x)$	standard normal density function

B Technical details and derivations

1. Tax system with a different tax rate on capital income

Our framework assumes that taxation is based on the net payoff $X_T - X_0$ from the overall investments that can be decomposed in the risky investment and a risk-free investment. Using the same tax rates for both sources of income as in our main analysis, thus, is a simplification. But, with a little bit of effort, this can be modified to different tax rates. In the modified framework, the risk-free return r is immediately taxed by a flat tax τ to result in an after-tax rate r_τ. The risky investment project is subject to a profit taxation where the after-tax value of the risky investments is

$$\tilde{X}_T = \begin{cases} \theta \cdot \big(1 + (1 - \tilde{\tau}_p) R_T\big) + (X_0 - \theta) \cdot (1 + r_\tau \cdot T)\,, & \text{if } R_T \geq 0 \\ \theta \cdot \big(1 + (1 - \tilde{\tau}_l) R_T\big) + (X_0 - \theta) \cdot (1 + r_\tau \cdot T)\,, & \text{if } R_T < 0 \end{cases}$$

where tax rates for the risky project are still given by equations (4) and (5), respectively. The firm's terminal wealth is then separated in two cases, one where the project achieves a positive return ($R_T \geq 0$) and is taxed at a (random) profit tax rate $\tilde{\tau}_p$ and the second case where the project return is negative ($R_T < 0$) and the firm receives a loss tax $\tilde{\tau}_l$ which includes loss offset restrictions. The expected utility in this framework can still be written in terms of a normal integral over the return R_T. Analogously, we obtain the utility with an ATR agreement $EU^{(2)}(\theta)$ and can perform the same analysis as in our main framework in Section 2. Thus, we expect that the main results of our main analysis still hold qualitatively if two different tax rates are applied to the return from the risky project and the risk-free government bond.

2. Expected utility and optimal investment under Procedure 1

For Procedure 1 (no ATR) and exponential utility $U(X) = -\frac{1}{\gamma} e^{-\gamma X}$, $\gamma > 0$, we can exploit the independence between $\tilde{\tau}_p$, $\tilde{\tau}_l$ and $\tilde{X}_T^{(1)}$ to write the expected utility as

$$\begin{aligned} EU^{(1)}(\theta) &= -\frac{1}{\gamma} \mathbb{E}\left[\exp\{-\gamma \tilde{X}_T^{(1)}\}\right] \\ &= -\frac{1}{\gamma} \mathbb{E}\Big[\, \mathbb{E}\big[\exp\{-\gamma(1 - \tilde{\tau}_p)(X_T - X_0)\} \cdot \mathbb{1}_{\{X_T \geq X_0\}} \end{aligned}$$

$$
\begin{aligned}
&\qquad\qquad + \exp\{-\gamma(1-\tilde{\tau}_l)(X_T - X_0)\} \cdot \mathbb{1}_{\{X_T < X_0\}} \,\Big|\, \tilde{\tau}_p,\, \tilde{\tau}_l \Big] \Big] \\
&= -\frac{1}{\gamma}\Bigg(d \cdot \mathbb{E}\Big[\exp\{-\gamma(1-\tau\cdot\delta_p)(X_T - X_0)\} \cdot \mathbb{1}_{\{X_T \geq X_0\}} \\
&\qquad + \exp\{-\gamma(1-\lambda\cdot\tau\cdot\delta_l)(X_T - X_0)\} \cdot \mathbb{1}_{\{X_T < X_0\}} \Big] \\
&\quad + (1-d)\cdot \mathbb{E}\Big[\exp\{-\gamma(1-\tau)(X_T - X_0)\} \cdot \mathbb{1}_{\{X_T \geq X_0\}} \\
&\qquad + \exp\{-\gamma(1-\lambda\cdot\tau)(X_T - X_0)\} \cdot \mathbb{1}_{\{X_T < X_0\}} \Big] \Bigg),
\end{aligned}
$$

where, in the first step, we have used property of iterated expectation. In the second step, we exploit the independence between $\tilde{\tau}_p$, $\tilde{\tau}_l$ and X_T. Due to the normal distribution of X_T, combined with exponential utility, it is possible to compute the above expected utility in closed form. In particular, for $g_1(a,b)$ and $g_2(a,b)$ as introduced in equations (10) and (11), we obtain as desired

$$
\begin{aligned}
&= -\frac{d}{\gamma}\big(\exp\{\gamma(1-\delta_p\tau)X_0\} \cdot g_1(X_0, \tau\delta_p) + \exp\{\gamma(1-\lambda\tau\delta_l)X_0\} \cdot g_2(X_0, \lambda\tau\delta_l)\big) \\
&\quad - \frac{1-d}{\gamma}\big(\exp\{\gamma(1-\tau)X_0\} \cdot g_1(X_0, \tau) + \exp\{\gamma(1-\lambda\tau)X_0\} \cdot g_2(X_0, \lambda\tau)\big)\,.
\end{aligned}
$$

Note that the expected utility is a function of investment amount θ which is hidden in the expected value and variance of the firm's time-T wealth X_T , that is in the functions $g_1(a,b)$ and $g_2(a,b)$. To determine the optimal investment amount, we shall take the first order derivative of the expected utility with respect to θ. From equations (2) and (3) it is easy to verify that

$$
\frac{\partial}{\partial\theta}\frac{\mathbb{E}[X_T] - (1+r\cdot T)X_0}{\sqrt{\operatorname{Var}[X_T]}} = \frac{\partial}{\partial\theta}\frac{1+\mu T - (1+r\cdot T)}{\sigma\sqrt{T}} = 0\,, \tag{18}
$$

$$
\frac{\partial\mathbb{E}[X_T]}{\partial\theta} = 1+\mu T - (1+r\cdot T) = \frac{1}{\theta}\big(\mathbb{E}[X_T] - (1+r\cdot T)X_0\big)\,, \tag{19}
$$

$$
\frac{\partial\operatorname{Var}[X_T]}{\partial\theta} = 2\sigma^2\theta T = \frac{2}{\theta}\operatorname{Var}[X_T]\,, \tag{20}
$$

$$
\frac{\partial\sqrt{\operatorname{Var}[X_T]}}{\partial\theta} = \sigma\sqrt{T} = \frac{1}{\theta}\sqrt{\operatorname{Var}[X_T]}\,, \tag{21}
$$

$$
\frac{\partial\frac{1}{\sqrt{\operatorname{Var}[X_T]}}}{\partial\theta} = -\frac{1}{\sigma\theta^2\sqrt{T}} = -\frac{1}{\theta}\frac{1}{\sqrt{\operatorname{Var}[X_T]}}\,. \tag{22}
$$

Referring to the difference of the two equations, (18)−(22), we can use the product rule to obtain

$$\begin{aligned}
\frac{\partial g_1(a,b)}{\partial\theta} &= \left(-\frac{1}{\theta}\Big(\gamma(1-b)\big(\mathbb{E}[X_T]-(1+r\cdot T)X_0\big)-\gamma^2(1-b)^2\operatorname{Var}[X_T]\Big)\right)\cdot g_1(a,b)\\
&\quad+\exp\left\{-\gamma(1-b)\mathbb{E}[X_T]+\frac{1}{2}\gamma^2(1-b)^2\operatorname{Var}[X_T]\right\}\\
&\quad\cdot\varphi\left(-\frac{a-\mathbb{E}[X_T]}{\sqrt{\operatorname{Var}[X_T]}}-\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right)\cdot\frac{1}{\theta}\left(\frac{a-(1+r\cdot T)X_0}{\sqrt{\operatorname{Var}[X_T]}}-\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right),\\
\frac{\partial g_2(a,b)}{\partial\theta} &= \left(-\frac{1}{\theta}\Big(\gamma(1-b)\big(\mathbb{E}[X_T]-(1+r\cdot T)X_0\big)-\gamma^2(1-b)^2\operatorname{Var}[X_T]\Big)\right)\cdot g_2(a,b)\\
&\quad+\exp\left\{-\gamma(1-b)\mathbb{E}[X_T]+\frac{1}{2}\gamma^2(1-b)^2\operatorname{Var}[X_T]\right\}\\
&\quad\cdot\varphi\left(\frac{a-\mathbb{E}[X_T]}{\sqrt{\operatorname{Var}[X_T]}}+\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right)\cdot\frac{1}{\theta}\left(-\frac{a-(1+r\cdot T)X_0}{\sqrt{\operatorname{Var}[X_T]}}+\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right),
\end{aligned}$$

where $\varphi(x) = 1/\sqrt{2\pi}e^{-x^2/2}$ denotes the density function of the standard normal distribution. Finally, the first derivative of the expected utility with respect to θ is given by

$$\begin{aligned}
\frac{\partial EU^{(1)}(\theta)}{\partial\theta} &= -\frac{d}{\gamma}\left(\exp\{\gamma(1-\delta_p\tau)X_0\}\frac{\partial g_1(X_0,\tau\delta_p)}{\partial\theta}+\exp\{\gamma(1-\lambda\tau\delta_l)X_0\}\frac{\partial g_2(X_0,\lambda\tau\delta_l)}{\partial\theta}\right)\\
&\quad-\frac{1-d}{\gamma}\left(\exp\{\gamma(1-\tau)X_0\}\frac{\partial g_1(X_0,\tau)}{\partial\theta}+\exp\{\gamma(1-\lambda\tau)X_0\}\frac{\partial g_2(X_0,\lambda\tau)}{\partial\theta}\right).
\end{aligned}$$

This shows that $EU^{(1)}(\theta)$ is continuous and differentiable in θ. We can further show that the expected utility $EU^{(1)}(\theta)$ is strictly concave in θ. This then confirms that there is a unique θ that maximizes $EU^{(1)}(\theta)$ on the compact interval $[0, X_0]$. The optimal θ is either one of the edge points $\theta = 0$ or $\theta = X_0$ or is determined such that

$$\frac{\partial EU^{(1)}(\theta)}{\partial\theta} = 0\,. \tag{23}$$

To show that $EU^{(1)}(\theta)$ is strictly concave in θ, we represent the terminal wealth X_T in equation (1) in terms of a standard normal random variable ϵ as follows. $X_T \sim (1 + r\cdot T)X_0 + \theta(\tilde{\mu} + \tilde{\sigma}\cdot\epsilon)$

with $\tilde{\mu} := (\mu - r)T$ and $\tilde{\sigma} := \sigma\sqrt{T}$. We can then write

$$g(\theta) := EU^{(1)}(\theta) = \mathbb{E}\big[f\big((1 + r \cdot T)X_0 + \theta(\tilde{\mu} + \tilde{\sigma} \cdot \epsilon)\big)\big]$$

for a function

$$f(x) := \begin{cases} d \cdot U\big((1 - \delta_p\tau)(x - X_0)\big) + (1 - d) \cdot U\big((1 - \tau)(x - X_0)\big), & x \geq X_0 \\ d \cdot U\big((1 - \lambda\delta_l\tau)(x - X_0)\big) + (1 - d) \cdot U\big((1 - \lambda\tau)(x - X_0)\big), & x < X_0 \end{cases} \tag{24}$$

The utility function $U(x) = -\frac{1}{\gamma}e^{-\gamma x}$ is obviously strictly concave implying that $f(x)$ is also strictly concave for $x > X_0$ and $x < X_0$. Looking at the slope at $X = x_0$, we can use that the average tax in the profit domain $\mathbb{E}[\tilde{\tau}_p]$ does not exceed the average tax in the loss domain $\mathbb{E}[\tilde{\tau}_l]$ to deduce that

$$f'(X_0-) = d \cdot (1 - \delta_p\tau) + (1 - d) \cdot \tau = \mathbb{E}[\tilde{\tau}_p] \leq \mathbb{E}[\tilde{\tau}_l] = d \cdot (1 - \lambda\delta_l\tau) + (1 - d) \cdot \lambda \cdot \tau = f'(X_0+)\,.$$

This shows that $f(x)$ is strictly concave in x, that is for any $x, y \in \mathbb{R}^+$ and $\alpha \in [0, 1]$, it holds that: $f(\alpha x + (1 - \alpha)y) > \alpha \cdot f(x) + (1 - \alpha) \cdot f(y)$. We can use this to show that $g(\theta)$ is strictly concave in θ. Choose $\theta_1, \theta_2 \in [0, X_0]$ arbitrary to get

$$\begin{aligned} g(\alpha\theta_1 + (1 - \alpha)\theta_2) &= \mathbb{E}\big[f\big((1 + r \cdot T)X_0 + (\alpha\theta_1 + (1 - \alpha)\theta_2)(\tilde{\mu} + \tilde{\sigma} \cdot \epsilon)\big)\big] \\ &= \mathbb{E}\Big[\mathbb{E}\big[f\big((1 + r \cdot T)X_0 + (\alpha\theta_1 + (1 - \alpha)\theta_2)(\tilde{\mu} + \tilde{\sigma} \cdot \epsilon)\big) \,\big|\, \epsilon\big]\Big] \\ &> \mathbb{E}\Big[\mathbb{E}\big[\alpha \cdot f\big((1 + r \cdot T)X_0 + \theta_1(\tilde{\mu} + \tilde{\sigma} \cdot \epsilon)\big) + (1 - \alpha) \cdot f\big(e^{rT}X_0 + \theta_2(\tilde{\mu} + \tilde{\sigma} \cdot \epsilon)\big) \,\big|\, \epsilon\big]\Big] \\ &= \alpha \cdot g(\theta_1) + (1 - \alpha) \cdot g(\theta_2)\,. \end{aligned}$$

The concavity of the objective function $g(\theta)$ in θ proves the uniqueness and existence of a solution θ^* of equation (8).

3. Expected utility and optimal investment under Procedure 2 (ATR)

For Procedure 2 with the advance tax rule (ATR), we can follow similar derivations as for Procedure 1 and obtain the expected utility as follows

$$
\begin{aligned}
EU^{(2)}(\theta) &= -\frac{1}{\gamma}\mathbb{E}\left[\exp\{-\gamma \tilde{X}_T^{(2)}\}\right] \\
&= -\frac{1}{\gamma}e^{\gamma F}\mathbb{E}\Big[\exp\{-\gamma(1-\tau\eta_p)(X_T - X_0)\}\cdot \mathbb{1}_{\{X_T \geq X_0\}} \\
&\qquad\qquad + \exp\{-\gamma(1-\lambda\tau\eta_l)(X_T - X_0)\}\cdot \mathbb{1}_{\{X_T < X_0\}}\Big] \\
&= -\frac{e^{\gamma F}}{\gamma}\Big(\exp\{\gamma(1-\tau\eta_p)X_0\}\cdot g_1(X_0,\tau\eta_p) + \exp\{\gamma(1-\lambda\tau\eta_l)X_0\}\cdot g_2(X_0,\lambda\tau\eta_l)\Big)\,.
\end{aligned}
$$

The first derivative of the expected utility with respect to θ is given by

$$
\frac{\partial EU^{(2)}(\theta)}{\partial\theta} = -\frac{e^{\gamma F}}{\gamma}\left(\exp\left\{\gamma(1-\tau\eta_p)X_0\right\}\frac{\partial g_1(X_0,\tau\eta_p)}{\partial\theta} + \exp\left\{\gamma(1-\lambda\tau\eta_l)X_0\right\}\frac{\partial g_2(X_0,\lambda\tau\eta_l)}{\partial\theta}\right)\,.
$$

The optimal θ^{**} is either 0, X_0 or determined such that

$$
\frac{\partial EU^{(2)}(\theta)}{\partial\theta} = 0\,. \tag{25}
$$

We can argue similar to the proof of Proposition 2 that there exists a unique solution θ^{**} of equation (8).

4. The impact of the tax loss offset parameter λ on optimal θ^{**}

Proof. Part (a) is obvious, since an increase in λ increases the net payoff and thus also the utility. Part (b): Let us first look at θ^{**}. If the short-selling constraint is binding and $\theta^{**} = X_0$, an increase in λ will not further increase the optimal amount as it is at its maximum already $\theta \in [0,\, X_0]$). Consider next the case where $\theta^{**} < X_0$. We know that for $\theta = \theta^{**}$, $\frac{\partial EU^{(2)}}{\partial\theta} = 0$. Both $EU^{(1)}(\theta)$ and $EU^{(2)}(\theta)$ are strictly concave in θ, see Appendix B, 2. This means that the second derivative $\partial^2 EU^{(2)}/\partial\theta^2$ is negative. To prove that the optimal amount θ^{**} is decreasing in λ, a differentiation of the first-order-condition gives:

$$
\frac{\partial\theta^{**}}{\partial\lambda} = -\frac{\partial^2 EU^{(2)}/(\partial\theta\,\partial\lambda)}{\partial^2 EU^{(2)}/\partial\theta^2}\,.
$$

Thus, to show that the amount θ^* is increasing in λ (i.e. $\partial\theta^{**}/\partial\lambda > 0$), we need to show that:

$$\frac{\partial^2 EU^{(2)}}{\partial\theta\,\partial\lambda} > 0\,.$$

Differentiating $\partial EU^{(2)}/\partial\theta$ from Appendix B, 3., w.r.t. λ,

$$\frac{\partial^2 EU^{(2)}}{\partial\theta\,\partial\lambda} = -\frac{e^{\gamma F}}{\gamma}\, e^{\gamma(1-\lambda\tau\eta_l)X_0}\left[-\gamma\tau\eta_l X_0\,\frac{\partial g_2(X_0,\lambda\tau\eta_l)}{\partial\theta} + \frac{\partial^2 g_2(X_0,\lambda\tau\eta_l)}{\partial\theta\,\partial\lambda}\right]. \tag{A1}$$

This is positive if and only if the following expression is negative:

$$\triangle = \frac{\partial^2 g_2(X_0,\lambda\tau\eta_l)}{\partial\theta\,\partial\lambda} - \gamma\tau\eta_l\cdot X_0\,\frac{\partial g_2(X_0,\lambda\tau\eta_l)}{\partial\theta}\,.$$

From the definition $g_2(X_0,\lambda\tau\eta_l) = \mathbb{E}\big[e^{-\gamma(1-\lambda\tau\eta_l)X_T}\cdot\mathbb{1}_{\{X_T<X_0\}}\big]$, we get that:

$$\begin{aligned}\frac{\partial g_2(X_0,\lambda\tau\eta_l)}{\partial\lambda} &= \mathbb{E}\Big[\mathbb{E}\Big[\frac{\partial}{\partial\lambda}e^{-\gamma(1-\lambda\tau\eta_l)X_T}\cdot\mathbb{1}_{\{X_T<X_0\}}\,\Big|\,X_T\Big]\Big]\\ &= \gamma\tau\eta_l\cdot\mathbb{E}\big[X_T\cdot e^{-\gamma(1-\lambda\tau\eta_l)X_T}\cdot\mathbb{1}_{\{X_T<X_0\}}\big] > 0\,.\end{aligned}$$

In Appendix B, 2., we have shown that $X_T \sim (1+r\cdot T)X_0 + \theta(\tilde\mu+\tilde\sigma\cdot\epsilon)$ with $\tilde\mu := (\mu-r)T$ and $\tilde\sigma := \sigma\sqrt{T}$ and a standard normal random variable ϵ. We can use this to get:

$$\begin{aligned}\frac{\partial^2 g_2(X_0,\lambda\tau\eta_l)}{\partial\theta\,\partial\lambda} &= \gamma\tau\eta_l\cdot\mathbb{E}\Big[X_T\cdot\frac{\partial}{\partial\theta}e^{-\gamma(1-\lambda\tau\eta_l)X_T}\cdot\mathbb{1}_{\{X_T<X_0\}}\Big]\\ &\qquad + \mathbb{E}\Big[\frac{X_T-(1+r\cdot T)X_0}{\theta}e^{-\gamma(1-\lambda\tau\eta_l)X_T}\cdot\mathbb{1}_{\{X_T<X_0\}}\Big]\end{aligned}$$

This helps us to show that:

$$\begin{aligned}\triangle &= \frac{\partial^2 g_2(X_0,\lambda\tau\eta_l)}{\partial\theta\,\partial\lambda} - \gamma\tau\eta_l\cdot X_0\,\frac{\partial g_2(X_0,\lambda\tau\eta_l)}{\partial\theta}\\ &= \lambda\tau\eta_l\cdot\Big(\mathbb{E}\Big[\underbrace{(X_T-X_0)}_{<0,\text{ since }X_T<X_0}\cdot\frac{\partial}{\partial\theta}e^{-\gamma(1-\lambda\tau\eta_l)X_T}\cdot\mathbb{1}_{\{X_T<X_0\}}\Big]\\ &\qquad + \mathbb{E}\Big[\underbrace{\frac{X_T-(1+r\cdot T)X_0}{\theta}}_{<0,\text{ since }X_T<X_0}e^{-\gamma(1-\lambda\tau\eta_l)X_T}\cdot\mathbb{1}_{\{X_T<X_0\}}\Big]\Big) < 0\,,\end{aligned}$$

using that $\frac{\partial g_2(X_0,\lambda\tau\eta_l)}{\partial\theta} = \mathbb{E}\big[\frac{\partial}{\partial\theta}e^{-\gamma(1-\lambda\tau\eta_l)X_T}\cdot\mathbb{1}_{\{X_T<X_0\}}\big] > 0$. □