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Advance Tax Rulings for Risky Investment under Multi-Dimensional Tax Uncertainty

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Abstract: In this study, we use advance tax rulings (ATR) to investigate the impact of fee-based tax certainty on risky investment decisions of a firm under both cash flow and tax uncertainty. We model and analyze the multi-dimensional nature of tax uncertainty from tax reforms and tax audits in expected tax rates, tax bases, and in conjunction with loss offset restrictions. A tax au-thority can provide tax certainty by offering ATRs and charging an ATR fee. The fee imposes costs on firms. We determine the critical ATR fee range in which the ATR is acceptable for both the firm and tax authority. Gener-ally, we find the ATR allows the firm to take on riskier investments. If the ATR is employed in an environment with a generous tax loss offset policy, the ATR's inducement effect on risky investments is even strengthened. We identify settings in which the tax authority is willing to charge zero or even negative ATR fees. Negative fees can be interpreted as enhanced services to taxpayers that reduce taxpayers' compliance costs. Surprisingly, we find that an ATR is particularly effective for firms with low risk aversion. Our findings suggest that ATRs can effectively fight tax uncertainty and stimulate invest-ment. However, their effectiveness crucially depends on tax system features such as loss offset restrictions and the ATR fee.

Keywords: advance tax ruling, cash flow uncertainty, loss offset provisions, optimal fee, optimal investment, tax uncertainty

JEL: G11, H25, M41, M42, M48

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1. Introduction

We analyze theoretically how the possibility of requesting fee-based advance tax rulings (ATRs) to mitigate tax uncertainty affects a firm's optimal investment decisions under both cash flow and tax uncertainty. Tax policy is often seen as an important regulatory tool to stimulate risky investment by creating an attractive environment (IMF 2017). While direct tax policy measures, such as loss offset provisions, are expected to encourage risk taking (Ljungqvist et al. 2017; Bethmann et al. 2018; Langenmayr and Lester 2018), politicians and tax practitioners often argue that tax uncertainty damps investment (IMF and OECD 2019; IMF and OECD 2018); see, for empirical evidence, for example, Edmiston 2004. Tax uncertainty is multi-dimensional, as it might stem from tax policy debates and reforms or unclear outcomes of tax audits. It also might be triggered by missing definitions for tax issues of new business models, unclear definitions, and attributions or judgments regarding the taxable income (profit or loss). Uncertain or unclear tax issues are likely to be challenged during a tax audit and can lead to a higher post-audit tax burden.

Due to increasing tax complexity and tax uncertainty over recent years, firms and governments are increasingly concerned (e.g., IMF and OECD 2019; Hoppe et al. 2021) and have called for remedies. As one particular remedy, many countries offer ATRs to mitigate tax uncertainty by providing upfront clarification of tax issues. Surprisingly, although difficulty in applying ambiguous tax laws and anticipating the consequences of a future tax audit has been highlighted in the literature (Mills et al. 2010; Lisowsky et al. 2013; Neuman et al. 2020), only a few scholarly contributions have examined the economic implications of rulings that are supposed to mitigate tax uncertainty (see Diller et al. 2017 for ATRs and De Waegenaere et al. 2007, and Becker et al. 2017 for advance pricing agreements). However, none of these studies accounts for the multi-dimensional character of tax uncertainty and and how it affects risky investments.

We aim to fill this void. We interpret tax uncertainty as tax payment uncertainty stemming from uncertainty in tax rates and tax bases, due to potential tax reforms, unclear tax issues, and tax audits, and focus on ATRs as a common policy tool to provide upfront clarification of unclear tax issues and mitigate multi-dimensional uncertainty.

In many countries, tax authorities must issue ATRs by law or offer them voluntarily to provide legal certainty (see Diller et al. 2017). An ATR is a statement provided by the tax authority or an independent council with respect to the tax treatment of a future transaction. The taxpayer can —at least to some extent—rely on the ATR. According to OECD (2019), ATRs are widely available. Of the 35 OECD countries, 33 allow ATRs.[1]

However, ATRs come at costs, so decision-makers must weigh these costs as well as benefits when evaluating the after-tax payoffs from risky investments with or without an ATR. We study how ATRs affect risky investments and how ATR fees should be set. To capture that the taxation of profits and losses differs across countries, we evaluate ATRs and their impact on investments under different tax schemes with different tax rates and loss offset regulations.

Whether a firm requests an ATR to eliminate a project's tax uncertainty depends on many factors, including uncertainty about the taxes and pre-tax cash flows, ATR fees, eligibility, and features of the tax system, such as loss offset provisions.[2] Many tax authorities charge a fee in return for offering taxpayers ATR certainty.[3] However, ATR fees vary considerably across countries and cases. In many countries, the fee depends proportionally on the hours the tax officers spend for the underlying issue. Some countries ask for upfront deposits; others have schemes with charges progressively increasing with hours above a threshold[4] or depend on the underlying

[1] The OECD report refers to data as of 2017. After 2017 three more countries joined the OECD (Lithuania 2018, Colombia 2020, Costa Rica 2021). In OECD (2019), these countries are included but labeled as non-OECD countries.

[2] For the relevance of losses in tax planning, see Dyreng et al. (2018), Henry and Sansing (2018).

[3] According to the OECD, 19 of the OECD countries and nine out 23 non-OECD countries charge ATR fees. See OECD (2019), Annex A, Table A.120.

[4] For details, see Canada Revenue Agency (2021); Internal Revenue Service (2016); Internal Revenue Service (2021); Inland Revenue Authority of Singapore (2019); South African Revenue Service (2020); Starkman (2010), p. 21.

turnover or value of dispute.[5] Some countries limit maximum fees, while others do not. Some countries also offer rulings free of charge.[6] To capture that a country can design and charge a fee, we model ATRs with and without fees and determine reasonable fee ranges that benefit both the taxpayer and tax authority. In this vein, we determine the crucial drivers of the fee, including the impact of tax system features, such as tax rate level and loss offset provisions.

Even though tax uncertainty often arises in conjunction with risky investments and ATRs aim to attenuate this uncertainty, the literature has largely overlooked the implications of fee-based ATRs for risky investments. To fill this gap, we consider R&D investments as an example of investments subject to both high cash-flow and tax uncertainty. R&D is innovative by nature, and thus not only are its future cash flows risky but, on top of this, routines for assessing its emerging tax issues are also lacking, which redoubles the uncertainty. For example, the OECD initiative to fight base erosion and profit shifting (BEPS, OECD 2013) has fueled tax reforms that affect R&D investments. The OECD recommendations on how to reform the taxation of digital businesses and intangibles (characteristics of many risky projects such as R&D projects) are loaded with unclear terms, resulting from compromises among the involved 139 countries.

These unclear definitions create high uncertainty, for example, regarding the existence, location, and value of an intangible asset, such as intellectual property (IP) from an R&D investment and the assignment of the resulting profits to a specific country and thus tax regime. To illustrate the magnitude of the underlying uncertainty, we refer to the example of the tax dispute between the US Internal Revenue Service and Facebook, which was heard in the US Tax Court. The parties disagreed about the valuation of intangible assets dating back to a tax return in 2010, with the disputed tax being more than US$9 billion (White 2020). Clearly, such conflicts impose significant uncertainty on corporations (Graham et al. 2014; Chen

[5]For details, see § 89 (5) 1 AO (German Fiscal Code), § 34 FGO (German Courts Fee Act); § 118 (10) BAO (Austrian Fiscal Code).

[6]For example, France; see Deloitte (2021)

et al. 2020; Neuman et al. 2020). In the face of budget challenges following the COVID 19 epidemic, competition on tax revenues across countries has further increased, suggesting the possibility of an increase in number and intensity of disputes between tax authorities and taxpayers (KPMG 2016; KPMG 2019; Nessa et al. 2020). As a consequence, corporations will have difficulties in anticipating the tax burden from their R&D investments without an upfront clarification from the tax authority. To enable corporations to appropriately integrate the expected tax implications of R&D investments in their decision-making, they might apply for an ATR.

We incorporate cash flow and tax uncertainty into a decision model developed by Diller et al. (2017). Also, we account for tax system features, such as tax rates and loss offset provisions. We assume that the profit from investments is subject to taxation and that loss offsets are offered. While profits are taxed at a proportional tax rate, many countries restrict loss offsets, at least to some extent. Under these restrictions, losses can be used to offset profits generated in previous or future periods (loss carrybacks and carryforwards), sometimes with additional limitations on amount and time. Hence losses are either fully offset, leading to an immediate tax refund at the profit tax rate, or refunded at a lower rate. Loss offset restrictions are common features of many countries' tax codes and prevent an immediate and complete tax refund. Providing generous loss offsets is often considered an investment incentive. Our approach allows us to capture the interaction of tax rates, loss offsets, and ATRs. This enables us to consider how this interaction affects the propensity to employ an ATR and make risky investments.

We analyze two research questions. First, we determine the optimal investment strategies of a company within two frameworks, i.e., without and with an ATR. This analysis enables us to investigate how the introduction of an ATR affects the optimal amount invested in a risky investment, such as R&D, and specifically whether the incorporation of the ATR induces firm risk taking. Second, we investigate how to set reasonable ATR fees. We study this question both from the firm's and tax authority's point of view, i.e. we determine the firm's maximum willingness to pay for the ATR and the lowest fee level acceptable for the tax authority. An agreement on an ATR between the two parties can only occur if the lowest acceptable fee is less than the highest willingness to pay of the firm. These analyses may help tax authorities establish the price range within which ATRs benefit themselves and firms.

Our model yields three main findings. First, we identify scenarios where an ATR with a zero fee can still be attractive for tax authorities. Beyond nonnegative fees, as deduced by Diller et al. (2017), we identify specific conditions under which tax authorities may even want to pay for an ATR. If, for example, tax authorities offer ATRs together with enhanced services to taxpayers, the resulting reduction in taxpayers' compliance costs can be interpreted as a negative ATR fee.

Second, we find interesting interactions between ATRs and the taxation of profits and losses that explain the occurrence of negative fees. If a generous tax loss offset policy is in place, the additional implementation of the ATR will enhance the incentive for firms to make risky investments. By offering tax certainty via an ATR, the tax authority has some discretion when fixing a relatively favorable or unfavorable taxation, that is, in our model, a low or high ATR tax rate. To make the ATR attractive, setting a relatively high (low) ATR profit (loss) tax rate must be offset with a zero or even negative ATR fee.

Third, we examine how sensitive firms' requests for ATRs are to their risk aversion. We thus extend the work of Diller et al. (2017), who assume that firms base their decisions on expected after-tax cash flows. In settings without or with the implementation of the ATR, we find a more risk-averse firm invests less into risky projects. However, we show the ATR influences investments by less risk-averse firms to a larger extent. With its tax rate ensured through the ATR, the less risk-averse firm will make riskier investments, expecting to be rewarded with a higher return. If a firm is more risk-averse, the ensured tax rates matter less, and the firm will per se pursue a less risky alternative investment. In this sense, we find that the reduction of risky investments caused by an increase in risk aversion is less substantial for the case with no ATR than for the case with an ATR. Surprisingly, we find that the willingness to pay for an ATR is non-monotone in the firm's risk aversion.

Our analysis of ATR fees reveals a complex relationship between ATRs, tax uncertainty, loss offset restrictions, and optimal investments. We thus contribute to two streams of literature. First, we contribute to theoretical studies on the ambiguous investment effects of taxes under cash flow uncertainty (e.g., Niemann and Sureth 2004; Alvarez and Koskela 2008; Gries et al. 2012; Kanniainen and Panteghini 2013) and tax uncertainty (Agliardi 2001; Niemann 2004; Niemann 2011). Several theoretical analyses find that uncertainty about tax policy hinders investments under specific conditions (see Sialm 2006; Niemann 2011). Further, anecdotal and empirical evidence indicates that tax uncertainty attenuates investments (Jacob et al. 2021) and risk-taking (Dharmapala and Hines 2009; Osswald and Sureth-Sloane 2020). However, all of these analyses abstract from the possibility of acquiring a tax uncertainty shield.

Therefore we also contribute to the literature on tax uncertainty shields. Among the few studies of ATRs, Givati (2009) analyzes taxpayers' strategic consideration of whether to request an ATR. He shows that the strategic disadvantages, such as increased inspection and risk of detection by tax examiners, outweigh benefits, such as avoidance of penalties. Relatedly, De Simone et al. (2013) study the implications of "enhanced relationship tax compliance programs" on optimal reporting and auditing. Under such a program, taxpayers disclose uncertain tax positions to the tax authority in exchange for a timely resolution. The authors identify settings under which these programs can reduce taxpayers' compliance cost and the tax authority's audit costs. Relatedly, research on advance pricing agreements, i.e., international bilateral or multilateral agreements between a taxpayer and a tax authority to reduce tax uncertainty, indicates that these agreements might increase compliance cost (De Waegenaere et al. 2007; Becker et al. 2017). However, none of these studies examines how tax-uncertainty shields and fee design affect firms' investment decisions. Diller et al. (2017) propose a discrete-time model to investigate an investor's willingness to pay for an ATR and how this translates into investments. Absent of other sources of uncertainty, like cash flow uncertainty, they assume that tax authorities integrate firms' reasoning into their decisions. They find that, in special cases, the optimal fee tax authorities should charge is prohibitively high, and thus firms will not request ATRs. Further, they show that ATRs under specific circumstances might foster investments.

All these studies focus on symmetric taxation of profits and losses. We extend their model and broaden the underlying scope of tax uncertainty. We introduce uncertain cash flows and account for risk aversion. Further, as risky investments are often characterized by (temporary) loss periods, we model loss offset restrictions to capture the typically asymmetric nature of the taxation of profits and losses and how this asymmetry interacts with tax uncertainty and the availability of an ATR. In a continuous-time setting, we allow for both stochastic pre-tax cash flows and tax uncertainty from tax reforms and tax audits. We investigate whether the implementation of an ATR will increase risky investments and increase the riskiness of firms' assets. We determine the "optimal fee" for the ATR, both from the firm's and the tax authority's point of view and thus identify conditions under which the tax authority and the firm can agree on an ATR. We further examine how this translates into risky investments.

We enhance extant optimal asset allocation models and exploit their continuous-time nature to derive closed-form solutions for an ATR design that benefits both the tax authority and the firm under multi-dimensional tax uncertainty.[7]

Our results also enhance the understanding of tax policy measures that may spur investments and highlight the crucial role of tax certainty and ATRs in optimal corporate investment strategies. We find that ATRs contribute to an attractive tax environment. Our model predicts that ATRs can alleviate the harms of tax uncertainty, foster investment, and amplify the risk-taking incentives of other tax

[7]In the optimal asset allocation literature, only the effect of stylized tax regulations, for example, proportional tax rates, has been studied. Yet the effects of more complex tax regulations with loss offset restrictions, tax uncertainty, and advance tax rulings have not been analyzed. So Seifried (2010) and Chen et al. (2019) study utility maximization problems of after-tax payoffs for a simple proportional tax rate on the return from banking and life insurance products. Our study advances the theoretical literature on optimal asset allocation by incorporating more sophisticated tax regulations and ATRs.

policy measures, such as loss offsets. However, these effects substantially depend on the chosen profit and loss tax rates and the ATR fee. These findings should help tax authorities better understand how firms respond to the taxation of risky investments and consequently how to assess and design tax policies. Specifically, our results can help them set reasonable fees for ATRs. Ultimately, as our study is theoretical with supporting numerical examples, our predictions on fee-dependent investment effects of tax certainty will need to be tested in future research. The implications are also interesting for firms, as our work suggests ways for them to make better asset allocation decisions by accounting for both taxes and legislative, administrative, and judicial tax uncertainty.

2. Model Setup, Investment Strategy, and Tax Procedures

We introduce the investment problem and the underlying financial market under two alternative tax procedures, that is, with and without an ATR. In a first step, an investor (a firm) wants to decide on how much to invest in a risky investment such as R&D. We solve for the optimal investment amount from the firm's perspective in both settings and, in a second step, determine reasonable fee ranges for the ATR and thereby incorporate the tax authority's decision on the ATR fee that anticipates the firm's decision making process.

2.1. Optimal Investment Amount

Investment strategy

We consider a time horizon $[0, T]$, $T < \infty$, and a fixed filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbb{P})$. The firm is endowed with an initial wealth X_0 and can invest in (1) a risk-free government bond $B_t := e^{rt}$ that accrues at the constant, risk-free interest rate r and (2) a risky investment project (e.g., R&D), whose value evolves according to the following stochastic process

$$\mathrm{d}S_t = S_t\left(\mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\right),\ S_0 > 0 \text{ is given,}$$

where μ, the rate of return on the risky project, and σ, the volatility with $\mu, \sigma > 0$, are constants and W denotes a standard one-dimensional Brownian motion under the *real world* measure $\mathbb{P}$. The Brownian motion is adapted to the filtration $\mathcal{F}$. We assume that the rate of return on the risky project exceeds the risk-free rate r, i.e., $\mu - r > 0$. The risk-free rate r can be interpreted as post-tax rate with a deterministic tax rate.

The value of the risky project is henceforth described by a log-normal distribution and assumed to be strictly positive, as assumed in the literature on the optimal asset allocation to achieve analytical solutions.[8] We denote the amount invested in the risky project by $\theta \in [0, X_0]$. In total, the firm's wealth evolves as

$$\begin{aligned}\mathrm{d}X_t &= \theta\,\frac{\mathrm{d}S_t}{S_t} + (X_t - \theta)\,r\,\mathrm{d}t \\ &= \big(rX_t + \big(\mu - r\big)\theta\big)\mathrm{d}t + \sigma\,\theta\mathrm{d}W_t, \quad X_0 > 0 \text{ is given.}\end{aligned} \tag{1}$$

Looking for the optimal amount invested in the risky project, we can solve the dynamics in equation (1) to obtain the terminal wealth X_T (see Appendix B.1)

$$X_T = e^{rT}X_0 + \theta\big(\mu - r\big)\int_0^T e^{r(T-s)}\mathrm{d}s + \sigma\theta\int_0^T e^{r(T-s)}\mathrm{d}W_s\,. \tag{2}$$

This implies that X_T is normally distributed with mean and variance

$$\mathbb{E}[X_T] = e^{rT}X_0 + \theta\big(\mu - r\big)\int_0^T e^{r(T-s)}\mathrm{d}s\,, \tag{3}$$

$$\mathrm{Var}[X_T] = \mathrm{Var}\left[\sigma\theta\int_0^T e^{r(T-s)}\mathrm{d}W_s\right] = \sigma^2\theta^2\int_0^T e^{2r(T-s)}\mathrm{d}s\,. \tag{4}$$

To determine the variance (equation (4)), we use the isometry-property of the Itô integral.

Taxation and ATR

[8]To obtain an analytical solution, Merton (1969) combines this log-normal assumption with preferences exhibiting constant absolute/relative risk aversion, and Chen et al. (2011) combine it with preferences showing symmetric asymptotic hyperbolic absolute risk aversion.

We assume that the return from the investments, i.e., the net payoff, $X_T - X_0$, may be positive (profit) or negative (loss) and is subject to different stochastic proportional tax rates depending on its profit or loss character. Here and in the following, the term "net" payoff denotes the terminal wealth less the initial investment.

If $X_T - X_0$ is positive, a tax rate $\tilde{\tau}_p$ is applied, leading to an after-tax payment of $(1 - \tilde{\tau}_p)(X_T - X_0)$. Assuming that the taxation of the return from the risky project is subject to tax uncertainty, we describe $\tilde{\tau}_p$ as a binary random variable

$$\tilde{\tau}_p = \begin{cases} \delta_p \cdot \tau & \text{with probability } d\,, \\ \tau & \text{with probability } 1 - d\,, \end{cases} \tag{5}$$

where $d \in (0, 1)$ is the probability of an adjustment of the assessed or anticipated tax payment due to a tax audit or reform. This adjustment translates into an increase in the tax rate by a tax risk multiplier $\delta_p \in (1, \frac{1}{\tau})$. For simplicity, in the following, we focus in our interpretations on tax audits as possible causes for such a tax adjustment. However, our analysis can be easily generalized towards tax adjustments due to tax reforms.

Typically, audit probabilities vary considerably within and across countries (Mendoza et al. (2017); Bachas et al. (2019)). A higher probability of a tax audit indicates higher tax uncertainty, i.e., higher δ_p. The tax risk multiplier δ_p is assumed to be independent of the pre-tax payment.

We analogously introduce a stochastic proportional tax rate for losses $\tilde{\tau}_l$, i.e., if the net payoff $X_T - X_0$ is negative, obtaining an after-tax payment of $(1 - \tilde{\tau}_l)(X_T - X_0)$. We describe $\tilde{\tau}_l$ as a binary random variable with δ_l the tax risk multiplier for losses

$$\tilde{\tau}_l = \begin{cases} \delta_l \cdot \lambda \cdot \tau & \text{with probability } d\,, \\ \lambda \cdot \tau & \text{with probability } 1 - d\,. \end{cases} \tag{6}$$

Here, the tax loss offset parameter $\lambda \in [0, 1]$ allows us to capture the asymmetric nature of taxation of profits and losses resulting from loss offset restrictions. The loss offset parameter λ takes the value $\lambda = 1$ if losses can be completely and immediately offset, implying an immediate tax refund at the tax rate τ (symmetric taxation of profits and losses). If the loss offset is restricted ($\lambda < 1$), unused losses can be carried forward until future profits allow for an offset or expire for tax loss offset purposes. If $\lambda = 0$, the tax loss cannot be offset at all. With probability $d \in (0, 1)$, a tax audit leads to a post-tax audit reduction of the tax offset by a tax risk multiplier $\delta_l \in (0, 1)$.[9]

With $\tilde{\tau}_p$ and $\tilde{\tau}_l$ and the parameters d, δ_p and δ_l, we operationalize tax uncertainty from a tax audit that might lead to a higher post-audit tax burden than originally assessed.[10] We summarize the after-tax payoff absent an ATR in Procedure 1:

Procedure 1 (no ATR): If the company decides not to request an ATR, the after-tax net payoff is given by

$$\tilde{X}_T^{(1)} = \begin{cases} (1 - \tilde{\tau}_p)(X_T - X_0), & \text{if } X_T \geq X_0\,, \\ (1 - \tilde{\tau}_l)(X_T - X_0), & \text{if } X_T < X_0\,. \end{cases} \tag{7}$$

By assumption and the introduction of the random tax rates $\tilde{\tau}_p$ and $\tilde{\tau}_l$ in equations (5) and (6), respectively, we observe that the tax audit leads to a lower after-tax net payoff $\tilde{X}_T^{(1)}$. In case of profits ($X_T \geq X_0$), with probability d, the firm must pay a higher tax at rate $\tau \cdot \delta_p > \tau$, and with probability $1 - d$, the project's tax rate is τ. In case of a loss ($X_T < X_0$), with probability d, the company receives a tax refund at a lower rate $\delta_l \cdot \lambda \cdot \tau < \lambda \cdot \tau$ on tax losses, and, with probability $1 - d$, the firm remains with a tax loss offset at rate $\lambda \cdot \tau$.

The firm can request an ATR, which provides shelter against tax uncertainty. After paying a fee F_0, the ATR ensures that the tax in the loss and profit domain are fixed ex ante. Thus, the random tax rate $\tilde{\tau}_p$ is replaced by a deterministic tax rate $\eta_p \cdot \tau$ and tax risk is resolved. The tax risk multiplier for profits η_p is $\in (1, \delta_p)$ and

[9] A reduction in tax loss offset might be due to a declared tax expenses that are not considered tax-deductible by the tax authority. The tax risk multiplier for losses δ_l can also be interpreted as a parameter that reflects the risk of having insufficient future cash flows for the loss offset.

[10] This definition of tax uncertainty is narrower than overall tax uncertainty. Hence our results reflect the lower bound of tax uncertainty, as uncertainty can stem from other reasons, too.

reflects the increase in the tax rate as a consequence of tax certainty provided to the firm by means of the ATR. This rate can be chosen by the tax authority such that the resulting combined tax rate lies in the interval between the two random outcomes of $\tilde{\tau}_p$ as described in equation (5). Correspondingly, the ATR also fixes the tax rate in the case of a loss at the level of η_l with $\lambda \cdot \tau$. Here $\eta_l \in (\delta_l, 1)$ is the tax risk multiplier for losses set by the tax authorities in the ATR such that the resulting deterministic combined tax rate lies in the interval as described by equation (6). The firm can choose whether it requests this ATR and pays the corresponding fee F_0 or whether the project's tax uncertainty remains. This decision depends on the tax rate multipliers η_p, η_l and the ATR fee F_0. Later, when we discuss reasonable choices for the ATR fee F_0, we will elaborate on the relationship between η_p, η_l and F_0. We summarize the after-tax payoff of Procedure 2 with ATR:

Procedure 2 (ATR): The firm decides to initiate an ATR by paying a fixed cost F_0 upfront. The main purpose of initializing an ATR is to avoid the tax uncertainty related to the random tax rates $\tilde{\tau}_p$ and $\tilde{\tau}_l$ for profits and losses, respectively. Hence, with an ATR, the firm pays for a fixed tax rate, which, however, can differ from τ. We assume that the ATR is perfect, i.e., completely eliminates tax uncertainty. The company can therefore replace its random tax rates $(\tilde{\tau}_p, \tilde{\tau}_l)$ by fixed tax rates $(\eta_p \cdot \tau, \eta_l \cdot \tau)$. Firms that initialize the ATR are seeking a less volatile after-tax net payoff at the maturity date T. The after-tax net payoff of the company at time T is as follows

$$\tilde{X}_T^{(2)} = \begin{cases} (1 - \eta_p \cdot \tau)(X_T - X_0) - F, & \text{if } X_T \geq X_0\,, \\ (1 - \eta_l \cdot \tau)(X_T - X_0) - F, & \text{if } X_T < X_0\,, \end{cases} \tag{8}$$

where we accrue the ATR fee F_0 at the risk-free rate r to obtain $F := F_0 e^{rT}$ at time T.[11] Note that the after-tax net payoff in the profit case $X_T \geq X_0$ can still be negative after deducing the fee F_0. Intuitively, in this case, the company has no incentives to

[11] For solvability reasons, we assume a non-tax-deductible fee. Explicitly modeling a tax-deductible fee does not allow for closed-form solutions. Therefore, we implicitly assume after-tax fees.

invest in the project with an ATR. We will elaborate on this when we determine the critical fee level where the firm is indifferent between Procedures 1 (no ATR) and 2 (ATR).

To determine the maximal expected utility of the firm for the two payoff schemes for a given fee, we assume that the firm's preferences can be described by an exponential utility function, that is, $U(X) = -\frac{1}{\gamma}e^{-\gamma X}$ with a risk-aversion coefficient $\gamma > 0$. In this sense, we extend Diller et al. (2017), where a firm's decisions are based on the expected after-tax cash flows. By contrast, we investigate the attractiveness of ATRs depending on the firm's risk aversion. Further, let us remark that the exponential utility is widely used and well justified in economics, finance, insurance and risk management; see, for example, Carmona (2009). As the after-tax net payoff can be negative, we cannot describe preferences by logarithmic or power utility that are exclusively defined for the positive real line. The firm wants to maximize the expected utility from the after-tax net payoff. The corresponding optimal investment problem of the firm under Procedures 1 and 2 is then given by

$$\max_{\theta \in [0, X_0]} EU^{(i)}(\theta) := \max_{\theta \in [0, X_0]} \mathbb{E}\left[-\frac{1}{\gamma}\exp\left\{-\gamma \tilde{X}_T^{(i)}\right\}\right], \; i = 1, 2, \tag{9}$$

$$\text{s.t. } X = (X_t)_{t \in [0,T]} \text{ follows equation (1).}$$

Using this objective function, we are assuming that the firm is interested in maximizing excess wealth, where the initial wealth X_0 is chosen as a basis for comparison. Note that maximizing the excess wealth in fact also relate to minimizing the probability that terminal wealth X_T falls below the initial wealth level X_0. The lower this probability, the higher the expected utility resulting from the excess wealth.

Obviously, here and in the following, the ATR is not a choice variable. By contrast, we use a two-step approach. In the first step we assume one of the two available tax procedures and then analyze the optimal investment problem under this assumption and reiterate this optimization for the other tax procedure. In the second step, we then compare the results of the two procedures to figure out under what conditions using the ATR is optimal.

For mathematical tractability, we have abstracted from the fact that tax rates on profits from risky investment projects like R&D might differ from tax rates on interest income. Accordingly, we also abstract from the resulting implications for the after-tax net payoff of the firm under such a tax system (e.g., a tax system with a lower proportional tax rate on interest income). In Appendix B.2, we show how the after-tax net payoff of the firm can be modified to this more granular tax framework. The optimization problem introduced below can still be solved semi-analytically. We expect the main results of the paper qualitatively hold in this setup.

Optimization

Using this two-step approach, we determine the expected utility of the firm and deduce the optimal amount to be invested in the risky project under either tax procedure, with and without the ATR. Detailed derivations can be found in Appendix B.3 and B.4.

Proposition 2.1 (Expected utility and optimal investment amount, Procedure 1) *Under Procedure 1, the expected utility as a function of the investment amount $\theta \in [0, X_0]$ is given by*

$$
\begin{aligned}
EU^{(1)}(\theta) &= -\frac{1}{\gamma}\mathbb{E}\left[\exp\{-\gamma \tilde{X}_T^{(1)}\}\right] \\
&= -\frac{d}{\gamma}\big(\exp\{\gamma(1-\delta_p\tau)X_0\}\cdot g_1(X_0,\tau\delta_p) + \exp\{\gamma(1-\lambda\tau\delta_l)X_0\}\cdot g_2(X_0,\lambda\tau\delta_l)\big) \\
&\quad - \frac{1-d}{\gamma}\big(\exp\{\gamma(1-\tau)X_0\}\cdot g_1(X_0,\tau) + \exp\{\gamma(1-\lambda\tau)X_0\}\cdot g_2(X_0,\lambda\tau)\big),
\end{aligned}
$$

where $g_1(a,b)$ and $g_2(a,b)$ are defined as

$$
\begin{aligned}
g_1(a,b) &:= \mathbb{E}\big[\exp\{-\gamma(1-b)X_T\}\cdot \mathbb{1}_{\{X_T\geq a\}}\big] = \exp\left\{-\gamma(1-b)\mathbb{E}[X_T] + \frac{1}{2}\gamma^2(1-b)^2\,\mathrm{Var}[X_T]\right\} \\
&\quad \cdot \Phi\left(-\frac{a-\mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}} - \gamma(1-b)\sqrt{\mathrm{Var}[X_T]}\right), \qquad (10) \\
g_2(a,b) &:= \mathbb{E}\big[\exp\{-\gamma(1-b)X_T\}\cdot \mathbb{1}_{\{X_T< a\}}\big] = \exp\left\{-\gamma(1-b)\mathbb{E}[X_T] + \frac{1}{2}\gamma^2(1-b)^2\,\mathrm{Var}[X_T]\right\}
\end{aligned}
$$

$$\cdot \Phi \left(\frac{a - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}} + \gamma(1-b)\sqrt{\mathrm{Var}[X_T]} \right) . \tag{11}$$

The optimal amount $\theta^* \in [0, X_0]$ solving equation (9) for Procedure $i = 1$ is either 0, X_0 or determined implicitly and uniquely by solving

$$\frac{\partial EU^{(1)}(\theta)}{\partial \theta} = 0 \,. \tag{12}$$

Proof: See Appendix B.3. We also provide an analytic expression for $\frac{\partial EU^{(1)}(\theta)}{\partial \theta}$. □

With Proposition 2.1 we derive an analytical expression for the expected utility of the firm as a function of the investment amount θ under the first tax procedure without an ATR. The optimal investment amount is determined by maximizing the expected utility of the firm. Proposition 2.2 derives the analogous expression for the tax procedure with an ATR.

Proposition 2.2 (Expected utility and optimal investment amount, Procedure 2) *Under Procedure 2, the expected utility as a function of the investment amount $\theta \in [0, X_0]$ is given by*

$$\begin{aligned} EU^{(2)}(\theta) &= -\frac{1}{\gamma}\mathbb{E}\left[\exp\{-\gamma \tilde{X}_T^{(2)}\}\right] \\ &= -\frac{e^{\gamma F}}{\gamma}\Big(\exp\{\gamma(1-\tau\eta_p)X_0\} \cdot g_1(X_0, \tau\eta_p) + \exp\{\gamma(1-\lambda\tau\eta_l)X_0\} \cdot g_2(X_0, \lambda\tau\eta_l)\Big) , \end{aligned}$$

where $F := F_0 e^{rT}$ and $g_1(a, b)$ and $g_2(a, b)$ are defined as in Proposition 2.1. The optimal amount $\theta^{**} \in [0, X_0]$ solving equation (9) for Procedure $i = 2$ is either 0, X_0 or determined implicitly and uniquely solving

$$\frac{\partial EU^{(2)}(\theta)}{\partial \theta} = 0 \,. \tag{13}$$

Proof: See Appendix B.4. We also provide an analytic expression for $\frac{\partial EU^{(2)}(\theta)}{\partial \theta}$. □

Due to the fixed and thus certain tax rate by means of adopting the ATR, the expected utility of the firm becomes simpler than under the first tax procedure. The fee F_0 is an important driver of the magnitude of the expected utility under Procedure 2. However, it does not affect the magnitude of the optimal investment amount θ, under this tax procedure, i.e., with the firm requesting an ATR.[12]

So far, we have determined the optimal investment amount under the two different tax procedures, assuming that the ATR fee level is given. In the following, we analyze how this fee can be reasonably set, considering the view point of both the firm and the tax authority. We determine the firm's maximal willingness to pay for the ATR and the lowest fee that is acceptable to the tax authority.

2.2. Reasonable ATR Fees

Firm's point of view

Whether the firm is willing to request an ATR depends on the firm's risk preferences, the tax system and tax uncertainty parameters, and the ATR fee in both tax procedures (i.e. the parameters d, δ_p, δ_l, λ, τ). In what follows, we assume that the model parameters are given, except the ATR fee F_0. We further assume that the firm always chooses the optimal investment amount θ^* (optimal amount under tax procedure 1) and θ^{**} (optimal amount under tax procedure 2) respectively. We determine the critical fee F_0^* that leads to identical utility levels for both tax procedures for the firm (firm's procedural indifference). The firm will request an ATR only if the expected utility is at least as high as absent an ATR. As the utility of the firm decreases monotonically in the fee, we conclude that the firm will request the ATR if $F_0 \leq F_0^*$; otherwise the firm chooses Procedure 1 with no ATR. Formally, the critical fee level F_0^* solves

$$EU^{(1)}(\theta^*) = EU^{(2)}(\theta^{**}) .$$

[12]This finding resembles the result observed in optimal insurance contracts. If an actuarially fair premium is charged for an insurance contract, the optimal insurance purchase of a risk-averse policyholder is full insurance. If instead a premium with a fixed surcharge is applied, the optimal insurance purchase of a risk-averse policyholder is either still full insurance or no insurance (which is the case if the surcharge is too high).

Applying Propositions 2.1 and 2.2, and $F_0 = Fe^{-rT}$, we obtain

$$F_0^* = \frac{e^{-rT}}{\gamma} \ln \frac{EU^{(1)}(\theta^*)}{h(\tau, \eta_p, \eta_l)}, \tag{14}$$

with $h(\tau, \eta_p, \eta_l) := -\frac{1}{\gamma}\Big[\exp\{\gamma(1-\tau\eta_p)X_0\} \cdot g_1(X_0, \tau\eta_p) - \exp\{\gamma(1-\lambda\tau\eta_l)X_0\} \cdot g_2(X_0, \lambda\tau\eta_l)\Big]$.

So far, we have taken the viewpoint of the firm. The firm chooses the ATR (Procedure 2) if the offered fee F_0 is less than the critical fee F_0^*. In this sense, F_0^* can be interpreted as the firm's maximal willingness to pay for the service provided by the ATR.

Tax authority's point of view

To compare the expected terminal tax revenue under both tax procedures, we assume that an ATR is offered if and only if the expected terminal tax and fee revenue from this firm is at least as high as in the case without an ATR (tax authority's procedural indifference). This assumption implies that the tax authority is risk neutral with respect to tax uncertainty. Further, we assume that the tax authority does not observe the risk aversion coefficient of the firm. Nor does it have the knowledge about the firm's specific investment strategies. However, the tax authority does know the distribution of the firm's terminal wealth at time T, i.e. X_T, based on which the tax authority can compute the expected terminal tax revenue.[13] For each distribution of X_T, the tax authority can compute a critical fee level. The critical fee level F_0^{**} is the lowest level the tax authority is willing to accept when offering an ATR for a given X_T distribution. At this level, the expected net revenue of the tax authority is zero as additional fee revenues from ATR compensate exactly the loss of tax revenue due to lower investments and profits of taxpayers in response to the ATR fee scheme. As a consequence, it is reasonable for the tax authority to provide the ATR for a given X_T distribution, if the fee is set higher than this critical level, i.e. $F_0 \geq F_0^{**}$.

Let us now derive the critical fee level of the tax authority F_0^{**}. As, in our setting,

[13]It is impossible in reality that the tax authority is able to observe both the firm's optimal investment strategy and the utility function. Thus, we exclude this by assumption. However, if we assumed that the tax authority is informed about the optimal investment strategies and the specific utility function of the firm, the authority could infer the risk aversion level of the firm.

X_T is coupled with the investment amount, we obtain the expected terminal wealth of the authority as a function of θ straightforwardly. For Procedure 1, the expected terminal tax revenue from this firm is

$$
\begin{aligned}
ER^{(1)}(\theta) &:= \tau \cdot (d \cdot \delta_p + (1-d)) \cdot \mathbb{E}\big[(X_T - X_0)\mathbb{1}_{\{X_T \geq X_0\}}\big] \\
&\qquad - \lambda\tau \cdot (d \cdot \delta_l + (1-d)) \cdot \mathbb{E}\big[(X_0 - X_T)\mathbb{1}_{\{X_T < X_0\}}\big] \qquad (15) \\
&= \tau \cdot (d \cdot \delta_p + (1-d)) \cdot \Bigg[\Big(1 - \Phi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big)\Big)\big(\mathbb{E}[X_T] - X_0\big) \\
&\qquad + \varphi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big) \cdot \sqrt{\mathrm{Var}[X_T]}\Bigg] - \lambda\tau \cdot (d \cdot \delta_l + (1-d)) \\
&\qquad \cdot \Bigg[\Phi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big) \cdot \big(X_0 - \mathbb{E}[X_T]\big) + \varphi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big) \cdot \sqrt{\mathrm{Var}[X_T]}\Bigg],
\end{aligned}
$$

where the standard normal density and cumulative distribution function are given by $\varphi(t) := \frac{1}{\sqrt{2\pi}}e^{-t^2/2}$ and $\Phi(x) := \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt$. In the last step, we use that the positive (respectively negative) part of $(X_T - X_0)$ follows a truncated normal distribution. Similarly, for Procedure 2, the expected terminal tax revenue is given by

$$
\begin{aligned}
ER^{(2)}(\theta) &= \eta_p\tau \cdot \mathbb{E}\big[(X_T - X_0)\mathbb{1}_{\{X_T \geq X_0\}}\big] - \lambda\eta_l\tau \cdot \mathbb{E}\big[(X_0 - X_T)\mathbb{1}_{\{X_T < X_0\}}\big] + F \\
&= \tau \cdot \Bigg[\eta_p\big(\mathbb{E}[X_T] - X_0\big) - \big(\eta_p - \lambda\eta_l\big) \cdot \Phi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big)\Big) \cdot \big(\mathbb{E}[X_T] - X_0\big) \\
&\qquad + (\eta_p - \lambda\eta_l) \cdot \varphi\Big(\frac{X_0 - \mathbb{E}[X_T]}{\sqrt{\mathrm{Var}[X_T]}}\Big) \cdot \sqrt{\mathrm{Var}[X_T]}\Bigg] + F\,. \qquad (16)
\end{aligned}
$$

As we have learned from Propositions 2.1 and 2.2, the firm invests different amounts in the risky project under the two tax procedures. The tax authority anticipates this behavior. Taking this into consideration, the critical fee level F_0^{**} is determined such that

$$ER^{(1)}(\theta^*) = ER^{(2)}(\theta^{**})\,. \qquad (17)$$

We therefore compute the fee level $F_0^{**} = e^{-rT}F_0$ satisfying equation (17). The parameters, such as δ_p, δ_l, η_p and η_l, are important drivers of the magnitude of F_0^{**}.

Overall, taking both the viewpoint of the firm and tax authority, an ATR is offered for an ATR fee $F_0 \geq F_0^{**}$. The firm accepts this fee according to its risk aversion and optimal investment amount if and only if this fee is lower than F_0^*. If $F_0^* < F_0^{**}$, an ATR will not be requested by the firm. Thus ATR fees with $F_0 \in [F_0^{**}, F_0^*]$, for $F_0^* \geq F_0^{**}$, are reasonable and beneficial to both the firm and the tax authority.

3. Numerical analyses

To gain an intuitive understanding of the results stated in section 2, Propositions 2.1 and 2.2, and the discussions about the critical fees, we conduct numerical analyses and therefore consider some parameters as given. In our baseline scenario, we assume

$$\mu = 0.06, \quad r = 0.02, \quad \sigma = 0.15, \quad \gamma = 0.02, \quad T = 5, \quad X_0 = 100, \quad \tau = 15\% \,.$$

The rate of return μ on the risky project and the volatility σ are chosen in line with a reasonable Sharpe ratio, i.e. a Sharpe ratio of $26.67\% \approx (\mu - r)/\sigma$. The Sharpe ratio describes the performance of a risky project in relation to a risk-free investment. Typically, a Sharpe ratio above 0.5 in the long run indicates great investment performance and is difficult to achieve, while a ratio between 0.1 and 0.3 is frequently considered reasonable and can be achieved more easily (see, e.g., Sharpe 1994). The choice of the risk aversion parameter γ is based on experimental studies.[14]

3.1. Optimal Investment Amount

Let us now analyze the optimal investment amount and the implications of both tax procedures, choosing the parameters set in our baseline scenario and six different choices of the parameters $(\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l,\ F_0,\ \tau)$. To illustrate the effect of tax uncertainty on the optimal investment amount in the case of R&D investments, in

[14] Experimental studies find reasonable parameters for the relative risk aversion (RRA) coefficient (e.g., Barsky et al. 1997). For the underlying exponential utility function, the absolute relative risk aversion (ARA) is constant and denoted by γ, while the RRA for exponential utility varies in time. For our numerical example, we deduce the ARA coefficient, i.e. γ, of the exponential utility from the observed RRA level in the existing literature by assuming that $\gamma \cdot X_0 =$ observed RRA levels. 65% of the data in Barsky et al. (1997) shows a RRA above 3.76, 24% below 2 and 12% between 2 and 3.76. Thus, we use $\gamma = 0.02$ and also use $\gamma = 0.03$ in the following.

Table 1, we choose a rather high tax uncertainty scenario with an audit probability varying from $d = 40\%$ to 60%. In the literature, audit probabilities vary considerably within and across countries (Mendoza et al. 2017; Bachas et al. 2019). There is evidence for extremely low audit probabilities with realistic magnitudes, ranging from 1% to 3% (Dhami and al Nowaihi 2007), as well as high audit rates, ranging from 20% to 50%, as reported, for example, by Collins and Plumlee 1991; Alm et al. 1992; Alm et al. 1993; Andreoni et al. 1998 and Bernasconi 1998. In Hoopes et al. (2012), the average estimated IRS audit rate for U.S. public firms between 1992 and 2008 is 29%, ranging from 0% to 55% with 19% in the 25th percentile to 37% in the 75th percentile of their data. Given that R&D is a matter with high-tax uncertainty, due to the inherent hard-to-value intangibles and transfer pricing issues, assuming even higher audit probabilities would be appropriate.

If there is an audit, the profit tax collected is assumed to be $\delta_p = 4$ (scenario (1)−(3)) and $\delta_p = 3$ (scenario (4)−(6)) times higher than originally declared.[15] To allow for a better interpretation of the resulting numbers, we display the certainty equivalent (CE) instead of the optimal utility level. The use of the CE makes the quantities easier to interpret, because the CE expresses the expected utility in monetary units instead of utility units. In our case, we can define the certainty equivalent as an amount of a certain capital that the firm shall receive as an equivalent for an uncertain terminal wealth arising from the R&D investment. As the argument of the utility is $X_T - X_0$, we define the certainty equivalent as follows

$$CE^{(1)} = -\frac{1}{\gamma} \ln\left(-\gamma EU^{(1)}(\theta^*)\right) + X_0\,, \quad CE^{(2)} = -\frac{1}{\gamma} \ln\left(-\gamma EU^{(2)}(\theta^{**})\right) + X_0\,,$$

where $CE^{(1)}$ and $CE^{(2)}$ give the certainty equivalents resulting from the case without

[15] These assumption account for the pronounced exposure of R&D investments to tax uncertainty. Disputes on what is tax deductible and transfer pricing may easily lead to a multiple times higher tax burden following a tax audit (e.g., 9 billion dollars of potential additional taxes for Facebook, see White (2020), and 13 billion dollars of expected additional taxes from a transfer pricing dispute of Coca-Cola, see Yee (2022). We conducted sensitivity analyses using, for example, $\delta_p = 1.5$ and found the same qualitative effects as discussed in the following. The general effect of tax uncertainty on optimal investment is, however, diminishing.

and with an ATR (Procedures 1 and 2), respectively. Table 1 illustrates the optimal investment amount and the certainty equivalent of the firm under the two tax procedures for various parameter combinations of $(\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l,\ F_0,\ \tau)$ and two choices of risk aversion γ. We assume an ATR fee F_0 inside the acceptance set $[F_0^{**}, F_0^{*}]$.[16]

< Insert Table 1 around here >

We observe three interesting effects. (a) Recall that, in case of an ATR, the random tax rate $\tilde{\tau}_p$ ($\tilde{\tau}_l$) becomes the fixed tax rate $\eta_p \cdot \tau$ ($\eta_l \cdot \tau$) if the company pays an additional ATR fee F_0. We find that the optimal investment amount in case of tax uncertainty (i.e. θ^*) is always lower than the optimal investment amount in case of an ATR (no tax risk, θ^{**}). We observe this result in all scenarios $(1)-(6)$ and for both risk aversion coefficients. This result suggests that here the ATR can foster risky investments for all levels of risk aversion. Because of the reduced tax uncertainty with an ATR, in the optimum, the firm increases its investments in R&D. This effect is quite intuitive, as eliminating tax uncertainty allows the firm to take on riskier projects and thus accept more cash-flow uncertainty. (b) Under both tax procedures, a more risk-averse firm (higher risk aversion parameter $\gamma = 0.03$) will invest less in the risky project than a less risk-averse firm ($\gamma = 0.02$). Moving from $\gamma = 0.02$ to 0.03, the firm reduces its holdings in the risky project. The magnitude of this decrease in the optimal investment amount does not differ substantially between the two tax procedures and is slightly higher in case of an ATR (i.e. θ^{**}) than in the case without ATR (i.e. θ^*). Surprisingly, an ATR has a larger impact for a less risk-averse firm. Through the "guaranteed" tax rates, the less risk-averse firm will undertake riskier investments, expecting to be rewarded with a higher return. If the firm is rather risk-averse, the guaranteed tax rates become less interesting to the firm, as it will per se pursue a less risky alternative investment. In this sense, the reduction of the optimal holding in a risky project caused by the increase in risk aversion is less substantial for the case with

[16]We provide a detailed discussions of reasonable ATR fees at the end of this section.

no ATR than for the case with an ATR. (c) We assume an audit probability increasing from $d = 40\%$ in scenario (1) to $d = 60\%$ in scenario (3). If there is no audit, the tax authority taxes profits at a rate τ. With probability d, an audit leads to a higher tax rate of $\delta_p \cdot \tau$. The average expected tax on profits, given by $\mathbb{E}[\tilde{\tau}_p] = d \cdot \delta_p \cdot \tau + (1 - d) \cdot \tau$, is increasing from scenario (1) to (3) and, in each scenario, is equal to η_p. We can easily see that the higher the tax rate multiplier η_p, the lower the certainty equivalent $CE^{(2)}$ for the firm resulting from the Procedure 2 with ATR (risk taking θ^{**}). In our numerical example, the magnitude of this decrease is very similar for $CE^{(1)}$ resulting from the Procedure 1 without an ATR (risk taking θ^{*}).

< Insert Table 2 around here >

Consistent with the literature (Ljungqvist et al. 2017, Bethmann et al. 2018, Langenmayr and Lester 2018), we find that more generous loss offset rules fuel risk-taking. Moreover, offering ATRs in an environment with generous loss offset provisions, further encourages risk-taking. To illustrate this amplifying effect, in Table 2, we vary the loss offset parameter λ, keeping the other parameters as in Table 1 and focusing on a uniform risk aversion level of $\gamma = 0.02$. Recall that λ is the tax offset parameter, where a λ value of 0 means there is no loss offset at all and a λ of 1 means a full tax offset. Compared to other parameters, the offset rate has a substantial influence on the firm's investment behavior. Two interesting observations emerge. (a) If the firm has no possibility to offset losses ($\lambda = 0$), it will invest less in the risky project. In comparison, the risky investment amount θ^{*} is comparably larger, if the loss offset parameter rises to 0.9 (c.f. Table 1). For the extreme case $\lambda = 1$ (a full loss offset), the optimal investment amount of the firm is highest. This finding is consistent with Ljungqvist et al. (2017), Langenmayr and Lester (2018) and Osswald and Sureth-Sloane (2020). Here, a tight loss offset provision "punishes" losses, as they are not fully tax-deductible. Then more conservative (less risky) investments with a lower probability of losses (lower θ) become more attractive. (b) The tax offset parameter λ also has a monotone effect on the optimal investment amount θ^{**} under Procedure 2 (with ATR). Compared to the case with no ATR, this effect is more pronounced; i.e., the increase in magnitude from θ^* to θ^{**} is more pronounced for $\lambda = 1$ than $\lambda = 0$. Specifically, for a given tax offset parameter λ, the application of ATR leads to a riskier investment, i.e. $\theta^{**} > \theta^*$. Implementing the ATR together with a generous tax offset policy, the encouraging-risky-investment effect will be further strengthened.

To analyze in more detail how tax uncertainty affects the optimal investment amount under the two tax regimes, we investigate how investments and tax uncertainty interact with each other and compare the results to the optimal asset allocation literature without taxes. In the case of an ATR with symmetric taxation ($\eta := \eta_p = \eta_l = 1$, $\lambda = 1$), the optimal investment decision under exponential utility is well studied. In this case, it turns out that the optimal investment amount θ^{**} is a function of the adjusted Sharpe ratio (ASR)

$$\mathrm{ASR} := \frac{\mu - r}{\sigma^2} \, ,$$

a result that is originally by Merton (1971). Adapted to our setting, a constant tax rate $\eta\tau$ leads to the optimal investment amount[17]

$$\theta^* = \theta^{**} = \frac{\mu - r}{\gamma\sigma^2(1-\tau)} \frac{\int_0^T e^{r(T-s)}\,\mathrm{d}s}{\int_0^T e^{2r(T-s)}\,\mathrm{d}s} = \frac{\mathrm{ASR}}{\gamma(1-\tau)} \frac{\int_0^T e^{r(T-s)}\,\mathrm{d}s}{\int_0^T e^{2r(T-s)}\,\mathrm{d}s} \, . \tag{18}$$

Using the parameter set of scenario (1) from Table 1, we obtain, for example, $\theta^{**} \approx 99.64$; i.e., the optimal investment strategy suggests investing almost all the initial wealth in the risky project. Figure 1 analyzes Procedure 2 (with ATR) in more detail. We want to see how an asymmetric taxation of profits and losses affects the optimal investment amount θ^{**}.

[17] This can be seen as follows: Abbreviating $\tilde{\mu} := (\mu - r)\int_0^T e^{r(T-s)}\,\mathrm{d}s$ and $\tilde{\sigma} := \sigma\sqrt{\int_0^T e^{2r(T-s)}\,\mathrm{d}s}$, we find for a constant and symmetric tax $\eta\tau$ that

$$\mathbb{E}\left[-\frac{1}{\gamma}\exp\left\{-\gamma(1-\tau)(X_T - X_0)\right\}\right] = -\frac{1}{\gamma}\exp\left\{\gamma(1-\tau)X_0 - \gamma(1-\tau)\tilde{\mu}\theta + \frac{1}{2}\gamma^2(1-\tau)^2\tilde{\sigma}^2\theta^2\right\} .$$

Taking first-order conditions with respect to θ, we realize that this objective is maximized if and only if the maximizer θ^{**} is given by equation (18).

$<$ Insert Figure 1 around here $>$

Figure 1 displays the optimal investment amount of Procedure 2 with ATR. The investment volatility σ is displayed at the x-axis. The rate of return μ on the risky project is adapted such that the adjusted Sharpe ratio is the same for each of the underlying investments considered. The left-hand side of Figure 1 presents the scenario with symmetric taxation ($\lambda = 1$). For the analysis, we look at different investment opportunities with identical adjusted Sharpe ratios; i.e., for different asset volatility σ, we choose pairs $(\mu, \sigma^2) = \left(\text{ASR} \cdot \sigma^2 + r, \sigma^2\right)$ and fix the adjusted Sharpe ratio ASR ≈ 1.777778 using the base case parameter set . We confirm the theoretical results of equation (18), i.e. that the optimal investment amount is constant for (μ, σ^2) with the same adjusted Sharpe ratio. A higher tax rate of the ATR (higher τ) leads to a higher optimal investment amount. The right-hand side of Figure 1 shows the effect of asymmetric taxation ($\lambda < 1$). We observe that the optimal investment amount is increasing in the loss-offset parameter λ. A smaller λ value means that the firm will end up with more net losses, if there are any, which forces it to take less risk. Further, if $\lambda < 1$, the optimal investment amount increases with increasing investment volatility. This can be explained by the fact that increasing the volatility – while keeping the ASR constant – leads to risky investments with a higher Sharpe ratio $(\mu - r)/\sigma$. A higher Sharpe ratio increases the likelihood of avoiding losses; that is why the optimal investment amount increases with the risk σ.

$<$ Insert Figure 2 around here $>$

Using scenario (1) from Table 1, Figure 2 presents the optimal investment amount in both tax regimes as a function of the asset volatility σ. Recall that we fix the adjusted Sharpe ratio and increase the project return according to the increased risk volatility. From the right graph, we observe that a higher investment return/risk leads to significantly higher optimal investment amounts θ^*, θ^{**}. This numerical result suggests that a simple relation between the ASR and the optimal investment amount, as in the constant and symmetric tax case of equation (18), is no longer true. We again observe that the ATR (straight lines) leads to an increased optimal investment amount, compared to the case without ATR (dashed lines). We further find that the optimal investment amounts θ^* and θ^{**} are decreasing in the company's risk aversion.[18] This result is intuitive in the case of symmetric taxation, where the optimal investment amount is inversely proportional to the risk aversion coefficient; see equation (18). In the case of loss offset restrictions ($\lambda < 1$), the effect of risk aversion on optimal investment is, however, not as pronounced. As analyzed in more detail in section 3, the reduced loss offset makes an increase in the investment amount and thus taking risk less attractive.

3.2. Reasonable ATR Fees

To numerically illustrate our theoretical findings from section 2 on the choice of reasonable ATR fees, we investigate when $F_0^{**} < F_0^*$ and hence under what conditions the tax authority and the firm agree on an ATR contract. Under the given set of assumptions, Table 3 provides the viewpoint of firms for various risk aversion levels γ and displays the resulting critical fee levels F_0^*. Recall that F_0^* is the maximum fee the firm is willing to pay for the ATR. Starting with a specific risk aversion level (in our example, $\gamma = 0.04$), we observe that a more risk averse firm (with a higher risk aversion level) is willing to pay more to eliminate tax uncertainty by requesting the ATR. The impact of risk aversion on the maximal willingness to pay F_0^* is not obvious, as can be seen in Table 3. We have to implicitly determine the critical fee by equating the optimal expected utility for the tax procedure with no ATR and the procedure with a fee-based ATR. As a higher risk aversion affects the expected utility for both procedures, its impact on the maximal willingness to pay F_0^* is not monotone and may lead to either an increase or decrease. This finding can be confirmed by the numerical results for F_0^* in Table 3, which are not monotone in the risk aversion level γ. In our example, if the firm is comparably less risk-averse, for example, $\gamma = 0.02$, a higher

[18]This additional analysis is available upon request from the authors.

maximal willingness to pay for the ATR results. Presumably, the optimal investment amount resulting from a lower γ is substantially larger, which can generate much more tax payments, if no ATR agreement is closed.

< Insert Tables 3 and 4 around here >

Table 4 takes the view of tax authorities and exhibits the critical fee level F_0^{**} for various combinations of tax rate multipliers η_p, η_l of the ATR. According to our considerations in section 2, the tax authority is willing to offer an ATR for any fee higher than F_0^{**}. Recall that $\eta_p \cdot \tau$ and $\eta_l \cdot \tau$ are the ultimately applied tax rates on profits and losses, respectively, as agreed on under the ATR. Both an increase in η_p and a decrease in η_l lead to higher average tax revenue for the tax authority. A higher tax on profits and a lower tax on losses both increase the overall tax payment. Consistently, Table 4 illustrates that the resulting critical fee F_0^{**} the tax authority requires is decreasing in η_p and $-\eta_l$.

The implementation of the ATR in case of a relatively low ATR profit tax η_p and a relatively high η_l requires a positive fee for the ATR. On the contrary, if the tax authority has already implemented a relatively high ATR profit tax (i.e. high η_p) and a relatively strict loss offset regulation (i.e. low η_l), the application of the ATR can be realized by a zero or negative fee. A negative F_0^{**} implies that any positive fee is acceptable for the tax authority and the authority is even willing to pay the firm (negative fee) if the firm requests the offered ATR. Such a negative fee can be interpreted as the tax authority's willingness to invest in the ATR, for example, in human resources or technology and thereby reduce taxpayers' compliance costs and make the ATR more attractive. Our results clarify that such a strategy benefits the tax authority for several sets of (fixed) tax rates. Note that, for Table 3, we assumed $\eta_p = 1.40$, $\eta_l = 0.90$. Under this set of assumptions, the firm's resulting maximal willingness to pay for the ATR is positive for all different risk aversion levels. For this combination of η_p and η_l, the minimum fee required by the tax authority is negative.

As a consequence, both the firm and the tax authority are ready to trade in the ATR. In this scenario, firms and the tax authority will always agree on the ATR for fees within the interval $[F_0^{**}, F_0^{*}]$.

Through the numerous numerical illustrations in this section, we confirm and quantify the main theoretical findings in section 2. If a firm chooses an ATR, it will allow the firm to take on riskier investments. More importantly, the tax authority can offer an acceptable ATR via different combinations of ATR fees on the one side and tax rates for profits and losses on the other. For example, the tax authority can choose a low profit tax and a positive ATR fee or a high profit tax and a negative ATR fee to achieve expected-revenue-neutral (i.e. zero expected revenue).

4. Conclusion

We investigate the impact of an ATR on firm's risky investments and how ATR fees can be set to benefit both the firm and the tax authority. Combining prior literature on taxation, advance tax rulings, and optimal asset allocation, we show that ATRs might foster risky investments and determine the reasonable fee range for an ATR. For most scenarios, such a range can be derived, and hence the ATR exchange between the firm and the tax authority can occur. Under specific conditions, for example, high ATR tax rates, a tax authority may even be willing to provide the ATR for a zero or even negative fee.

Further, we show that the firm's and the tax authority's decisions to agree on a fee-based ATR crucially depend on the loss offset policy and profit tax rate as well as the firm's risk aversion level. A generous tax loss offset amplifies the ATR's potential to encourage firm risk taking. We find that a firm with low risk aversion seeks a riskier investment than a more risk-averse firm. More interestingly, taking on the ATR influences a less risk-averse firm's investment behavior to a larger extent. Hence the implementation of the ATR will increase the riskiness of the projects, irrespective of the firm's risk aversion. However, this ATR-induced increase in riskiness is more pronounced for a less risk-averse firm.

Our model is limited by its assumptions. The return from the risky investments is assumed to follow relatively simple dynamics, the Black-Scholes setting with a constant volatility. In future research, more complex asset dynamics, moving beyond normally distributed log-returns and describing the evolution of the risky investments with a more general stochastic volatility model in the sense of Heston (1993) can be developed. A stochastic volatility model would allow researchers to generalize our model and capture risk patterns that can be often observed in financial markets, such as heavy tails, volatility clustering, and the smile of implied volatilities (see Tankov 2003). The incorporation of volatility risk intensifies the cash flow uncertainty. It would be interesting to investigate whether our findings about the impact of the ATR and other tax policies on asset allocation prevail in a more volatile market. Further, while we assumed exponential utility, i.e. constant absolute risk aversion preference, to describe the firm's risk aversion, future research should consider behavioral risk and loss attitudes, as known, for example, from cumulative prospect theory. This could better capture possible gambling by firms.

Although our analyses are conducted in a stylized model, our results enhance the understanding of ATR-induced investment effects and the corresponding combined effects of ATRs and loss offset regimes. The negative impact of tax uncertainty on risky investments can be attenuated or even reversed by appropriately designed ATRs. Our predictions on the fee-dependent investment effects of ATRs should be tested empirically. With such empirical analyses, our results can help tax authorities develop policies to mitigate tax uncertainty and encourage risky investment. Knowing how firms invest in response to ATRs under different tax regimes will help tax authorities design better ATRs. Our results also provide interesting insights for firms about whether to apply for ATRs and may help them make better asset allocation decisions by accounting for both taxes and legislative, administrative and judicial tax uncertainty.

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Appendix

A. Variables

< Insert Table 5 around here >

B. Technical details and derivations

B.1. Terminal wealth

Apply Itô's Lemma on the dynamics of equation (1) using the function $f(x) = e^{-rt} \cdot x$ to obtain

$$\mathrm{d}(e^{-rt}X_t) = \theta e^{-rt}\big((\mu - r)\mathrm{d}t + \sigma \mathrm{d}W_t\big) \,.$$

This implies that

$$e^{-rt}X_t = X_0 + \theta \int_0^t e^{-rs}(\mu - r)\mathrm{d}s + \theta\sigma \int_0^t e^{-rs}\mathrm{d}W_s \,,$$

which can be used to obtain equation (2), as desired.

B.2. Tax system with a different tax rate on capital income

Our framework assumes that taxation is based on the net payoff $X_T - X_0$ from the overall investments that can be decomposed in the risky project and a risk-free government bond. In various countries different tax rates apply to the return from the R&D investments and the return from government bonds. Using uniform tax rates for both sources of income as in our main analysis, thus, is a simplification. To clarify that our results also hold for other settings, we show and analyze how our framework can be enhanced for such a tax system with different tax rates. Recall the time-T value of the risky project

$$S_T = S_0 \exp\{R_T^S\}, \text{ with } R_T^S = \Big(\mu - \frac{\sigma^2}{2}\Big)T + \sigma W_T \,.$$

In the modified framework, the risk-free return r is immediately taxed by a flat tax τ The risky investment project is subject to a profit taxation where the after-tax value of the risky investments is

$$\tilde{S}_T = \begin{cases} S_0 \exp\{(1-\tilde{\tau}_p) R_T^S\} & \text{if } S_T \geq S_0 \\ S_0 \exp\{(1-\tilde{\tau}_l) R_T^S\} & \text{if } S_T < S_0 \,, \end{cases}$$

where tax rates for the risky project are still given by equations (5) and (6), respectively. With this slight modification, we can follow the same steps as in section 2. The firm's terminal wealth is then separated in two cases, one where the project achieves a positive return ($S_T \geq S_0$) and is taxed at a (random) profit tax rate τ_p and the second case where the project return is negative ($S_T < S_0$) and the company receives a loss tax τ_l including loss offset restrictions. Calculations similar to the ones in Appendix B.1 lead to the terminal firm value after tax

$$\tilde{X}_T = \begin{cases} e^{r_\tau T} X_0 + \theta\big(\mu(1-\tilde{\tau}_p) - r_\tau\big) \int\limits_0^T e^{r_\tau (T-s)} \mathrm{d}s + \sigma(1-\tilde{\tau}_p)\theta \int\limits_0^T e^{r_\tau(T-s)} \mathrm{d}W_s \,, & \text{if } S_T \geq S_0 \,, \\ e^{r_\tau T} X_0 + \theta\big(\mu(1-\tilde{\tau}_l) - r_\tau\big) \int\limits_0^T e^{r_\tau (T-s)} \mathrm{d}s + \sigma(1-\tilde{\tau}_l)\theta \int\limits_0^T e^{r_\tau(T-s)} \mathrm{d}W_s \,, & \text{if } S_T < S_0 \,, \end{cases}$$

where we abbreviate $r_\tau = r(1-\tau)$. We can still compute the expected utility rather easily in terms of an integral expression. Exploiting that ($S_T \geq S_0$) is equivalent to a positive return ($R_T^S \geq 0$), the utility can be written as an integral over the bivariate density $f(U,V)$ of $(U,V) = (R_T^S,\, \sigma\theta \int\limits_0^T e^{r_\tau(T-s)} \mathrm{d}W_s)$

$$\begin{aligned} EU^{(1)}(\theta) =& -\frac{1}{\gamma}\mathbb{E}\left[\exp\{-\gamma \tilde{X}_T\}\right] \\ =& -\frac{1}{\gamma}\Bigg[\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \exp\Big\{-\gamma(e^{r_\tau T}X_0 + \big(\mu(1-\tilde{\tau}_p) - r_\tau\big)c + (1-\tilde{\tau}_p)\cdot v)\Big\}\cdot \mathbb{1}_{\{u\geq 0\}} \cdot f(u,v)\,\mathrm{d}u\,\mathrm{d}v \\ &+ \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \exp\Big\{-\gamma(e^{r_\tau T}X_0 + \big(\mu(1-\tilde{\tau}_l) - r_\tau\big)c + (1-\tilde{\tau}_l)\cdot v)\Big\}\cdot \mathbb{1}_{\{u< 0\}} \cdot f(u,v)\,\mathrm{d}u\,\mathrm{d}v\Bigg], \end{aligned}$$

where we abbreviated $c = \theta \int\limits_0^T e^{r_\tau(T-s)} \mathrm{d}s$. Analoguously, we obtain the utility with an ATR agreement $EU^{(2)}(\theta)$ and can perform the same analysis as in our main framework in section 2. Note that the expected utility of the firm under both tax procedures can still be determined semi-analytically, i.e. in integral form. The same holds for the optimization problem. We expect that the main results of our main analysis still hold qualitatively if two different tax rates are applied to the return from the risky project and the risk-free government bond.

B.3. Expected utility and optimal investment under Procedure 1

For Procedure 1 and exponential utility $U(X) = -\frac{1}{\gamma}e^{-\gamma X}$, $\gamma > 0$, we can exploit the independence between $\tilde{\tau}_p$, $\tilde{\tau}_l$ and $\tilde{X}_T^{(1)}$ to write the expected utility as

$$
\begin{aligned}
EU^{(1)}(\theta) =& -\frac{1}{\gamma}\mathbb{E}\left[\exp\{-\gamma\tilde{X}_T^{(1)}\}\right] \\
=& -\frac{1}{\gamma}\mathbb{E}\Big[\mathbb{E}\big[\exp\{-\gamma(1-\tilde{\tau}_p)(X_T - X_0)\}\cdot\mathbb{1}_{\{X_T\geq X_0\}} \\
& \qquad + \exp\{-\gamma(1-\tilde{\tau}_l)(X_T - X_0)\}\cdot\mathbb{1}_{\{X_T< X_0\}}\,\big|\,\tilde{\tau}_p,\,\tilde{\tau}_l\big]\Big] \\
=& -\frac{1}{\gamma}\Bigg(d\cdot\mathbb{E}\Big[\exp\{-\gamma(1-\tau\cdot\delta_p)(X_T - X_0)\}\cdot\mathbb{1}_{\{X_T\geq X_0\}} \\
& \quad + \exp\{-\gamma(1-\lambda\cdot\tau\cdot\delta_l)(X_T - X_0)\}\cdot\mathbb{1}_{\{X_T< X_0\}}\Big] \\
& + (1-d)\cdot\mathbb{E}\Big[\exp\{-\gamma(1-\tau)(X_T - X_0)\}\cdot\mathbb{1}_{\{X_T\geq X_0\}} \\
& \quad + \exp\{-\gamma(1-\lambda\cdot\tau)(X_T - X_0)\}\cdot\mathbb{1}_{\{X_T< X_0\}}\Big]\Bigg),
\end{aligned}
$$

where, in the first step, we have used property of iterated expectation. In the second step, we exploit the independence between $\tilde{\tau}_p$, $\tilde{\tau}_l$ and X_T. Due to the normal distribution of X_T, combined with exponential utility, it is possible to compute the above expected utility in closed form. In particular, for $g_1(a,b)$ and $g_2(a,b)$ as introduced in equations (10) and (11), we obtain as desired

$$
\begin{aligned}
=& -\frac{d}{\gamma}\big(\exp\{\gamma(1-\delta_p\tau)X_0\}\cdot g_1(X_0,\tau\delta_p) + \exp\{\gamma(1-\lambda\tau\delta_l)X_0\}\cdot g_2(X_0,\lambda\tau\delta_l)\big) \\
& - \frac{1-d}{\gamma}\big(\exp\{\gamma(1-\tau)X_0\}\cdot g_1(X_0,\tau) + \exp\{\gamma(1-\lambda\tau)X_0\}\cdot g_2(X_0,\lambda\tau)\big)\,.
\end{aligned}
$$

Note that the expected utility is a function of investment amount θ which is hidden in the expected value and variance of X_T, that is in the functions $g_1(a,b)$ and $g_2(a,b)$. To determine the optimal investment amount, we shall take the first order derivative of the expected utility with respect to θ. From equations (3) and (4) it is easy to verify that

$$\frac{\partial}{\partial\theta}\frac{\mathbb{E}[X_T]-e^{rT}X_0}{\sqrt{\operatorname{Var}[X_T]}} = \frac{\partial}{\partial\theta}\frac{(\mu-r)\int_0^T e^{r(T-s)}\mathrm{d}s}{\sigma\sqrt{\int_0^T e^{2r(T-s)}\mathrm{d}s}} = 0\,, \tag{19}$$

$$\frac{\partial\mathbb{E}[X_T]}{\partial\theta} = (\mu-r)\int_0^T e^{r(T-s)}\mathrm{d}s = \frac{1}{\theta}\big(\mathbb{E}[X_T]-e^{rT}X_0\big)\,, \tag{20}$$

$$\frac{\partial\operatorname{Var}[X_T]}{\partial\theta} = 2\sigma^2\theta\int_0^T e^{2r(T-s)}\mathrm{d}s = \frac{2}{\theta}\operatorname{Var}[X_T]\,, \tag{21}$$

$$\frac{\partial\sqrt{\operatorname{Var}[X_T]}}{\partial\theta} = \sigma\sqrt{\int_0^T e^{2r(T-s)}\mathrm{d}s} = \frac{1}{\theta}\sqrt{\operatorname{Var}[X_T]}\,, \tag{22}$$

$$\frac{\partial\frac{1}{\sqrt{\operatorname{Var}[X_T]}}}{\partial\theta} = -\frac{1}{\sigma\theta^2\sqrt{\int_0^T e^{2r(T-s)}\mathrm{d}s}} = -\frac{1}{\theta}\frac{1}{\sqrt{\operatorname{Var}[X_T]}}\,. \tag{23}$$

Referring to the difference of the two equations, (19)−(23), we can use the product rule to obtain

$$\begin{aligned}
\frac{\partial g_1(a,b)}{\partial\theta} &= \left(-\frac{1}{\theta}\Big(\gamma(1-b)\big(\mathbb{E}[X_T]-e^{rT}X_0\big)-\gamma^2(1-b)^2\operatorname{Var}[X_T]\Big)\right)\cdot g_1(a,b)\\
&\quad+\exp\left\{-\gamma(1-b)\mathbb{E}[X_T]+\frac{1}{2}\gamma^2(1-b)^2\operatorname{Var}[X_T]\right\}\\
&\quad\cdot\varphi\left(-\frac{a-\mathbb{E}[X_T]}{\sqrt{\operatorname{Var}[X_T]}}-\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right)\cdot\frac{1}{\theta}\left(\frac{a-e^{rT}X_0}{\sqrt{\operatorname{Var}[X_T]}}-\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right)\,,\\
\frac{\partial g_2(a,b)}{\partial\theta} &= \left(-\frac{1}{\theta}\Big(\gamma(1-b)\big(\mathbb{E}[X_T]-e^{rT}X_0\big)-\gamma^2(1-b)^2\operatorname{Var}[X_T]\Big)\right)\cdot g_2(a,b)\\
&\quad+\exp\left\{-\gamma(1-b)\mathbb{E}[X_T]+\frac{1}{2}\gamma^2(1-b)^2\operatorname{Var}[X_T]\right\}\\
&\quad\cdot\varphi\left(\frac{a-\mathbb{E}[X_T]}{\sqrt{\operatorname{Var}[X_T]}}+\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right)\cdot\frac{1}{\theta}\left(-\frac{a-e^{rT}X_0}{\sqrt{\operatorname{Var}[X_T]}}+\gamma(1-b)\sqrt{\operatorname{Var}[X_T]}\right)\,,
\end{aligned}$$

where $\varphi(x)=1/\sqrt{2\pi}e^{-x^2/2}$ denotes the density function of the standard normal distribution. Finally, the first derivative of the expected utility with respect to θ is given by

$$\begin{aligned}
\frac{\partial EU^{(1)}(\theta)}{\partial\theta} &= -\frac{d}{\gamma}\left(\exp\{\gamma(1-\delta_p\tau)X_0\}\frac{\partial g_1(X_0,\tau\delta_p)}{\partial\theta}+\exp\{\gamma(1-\lambda\tau\delta_l)X_0\}\frac{\partial g_2(X_0,\lambda\tau\delta_l)}{\partial\theta}\right)\\
&\quad-\frac{1-d}{\gamma}\left(\exp\{\gamma(1-\tau)X_0\}\frac{\partial g_1(X_0,\tau)}{\partial\theta}+\exp\{\gamma(1-\lambda\tau)X_0\}\frac{\partial g_2(X_0,\lambda\tau)}{\partial\theta}\right).
\end{aligned}$$

This shows that $EU^{(1)}(\theta)$ is continuous and differentiable in θ. We can further show that the expected utility $EU^{(1)}(\theta)$ is strictly concave in θ. This then confirms that there is a unique θ that maximizes $EU^{(1)}(\theta)$ on the compact interval $[0, X_0]$. The optimal θ is either one of the edge points $\theta = 0$ or $\theta = X_0$ or is determined such that

$$\frac{\partial EU^{(1)}(\theta)}{\partial \theta} = 0\,. \tag{24}$$

To show that $EU^{(1)}(\theta)$ is strictly concave in θ, we represent the terminal wealth X_T in equation (2) in terms of a standard normal random variable ϵ as follows.
$X_T \sim e^{rT} X_0 + \theta(\tilde{\mu} + \tilde{\sigma} \cdot \epsilon)$ with $\tilde{\mu} := (\mu - r) \int_0^T e^{r(T-s)} \mathrm{d}s$ and $\tilde{\sigma} := \sigma \sqrt{\int_0^T e^{2r(T-s)} \mathrm{d}s}$.
We can then write

$$g(\theta) := EU^{(1)}(\theta) = \mathbb{E}\big[f\big(e^{rT} X_0 + \theta(\tilde{\mu} + \tilde{\sigma} \cdot \epsilon)\big)\big]$$

for a function

$$f(x) := \begin{cases} d \cdot U\big((1 - \delta_p \tau)(x - X_0)\big) + (1 - d) \cdot U\big((1 - \tau)(x - X_0)\big), & x \geq X_0 \\ d \cdot U\big((1 - \lambda \delta_l \tau)(x - X_0)\big) + (1 - d) \cdot U\big((1 - \lambda \tau)(x - X_0)\big), & x < X_0 \end{cases} \tag{25}$$

The utility function $U(x) = -\frac{1}{\gamma} e^{-\gamma x}$ is obviously strictly concave implying that $f(x)$ is also strictly concave for $x > X_0$ and $x < X_0$. Looking at the slope at $X = x_0$, we can use that the average tax in the profit domain $\mathbb{E}[\tilde{\tau}_p]$ does not exceed the average tax in the loss domain $\mathbb{E}[\tilde{\tau}_l]$ to deduce that

$$\begin{aligned} f'(X_0-) = d \cdot (1 - \delta_p \tau) + (1 - d) \cdot \tau = \mathbb{E}[\tilde{\tau}_p] \\ \leq \mathbb{E}[\tilde{\tau}_l] = d \cdot (1 - \lambda \delta_l \tau) + (1 - d) \cdot \lambda \cdot \tau = f'(X_0+)\,. \end{aligned}$$

This shows that $f(x)$ is strictly concave in x, that is for any $x, y \in \mathbb{R}^+$ and $\alpha \in [0, 1]$, it holds that: $f(\alpha x + (1 - \alpha) y) > \alpha \cdot f(x) + (1 - \alpha) \cdot f(y)$. We can use this to show that

$g(\theta)$ is strictly concave in θ. Choose $\theta_1, \theta_2 \in [0, X_0]$ arbitrary to get

$$
\begin{aligned}
g(\alpha\theta_1 + (1-\alpha)\theta_2) &= \mathbb{E}\big[f\big(e^{rT}X_0 + (\alpha\theta_1 + (1-\alpha)\theta_2)(\tilde{\mu} + \tilde{\sigma}\cdot\epsilon)\big)\big] \\
&= \mathbb{E}\Big[\mathbb{E}\big[f\big(e^{rT}X_0 + (\alpha\theta_1 + (1-\alpha)\theta_2)(\tilde{\mu} + \tilde{\sigma}\cdot\epsilon)\big) \mid \epsilon\big]\Big] \\
&> \mathbb{E}\Big[\mathbb{E}\big[\alpha\cdot f\big(e^{rT}X_0 + \theta_1(\tilde{\mu} + \tilde{\sigma}\cdot\epsilon)\big) + (1-\alpha)\cdot f\big(e^{rT}X_0 + \theta_2(\tilde{\mu} + \tilde{\sigma}\cdot\epsilon)\big) \mid \epsilon\big]\Big] \\
&= \alpha\cdot g(\theta_1) + (1-\alpha)\cdot g(\theta_2)\,.
\end{aligned}
$$

The concavity of the objective function $g(\theta)$ in θ proves the uniqueness and existence of a solution θ^* of equation (9).

B.4. Expected utility and optimal investment under Procedure 2

For Procedure 2 with the advance tax rule (ATR), we can follow similar derivations as for Procedure 1 and obtain the expected utility as follows

$$
\begin{aligned}
EU^{(2)}(\theta) &= -\frac{1}{\gamma}\mathbb{E}\left[\exp\{-\gamma\tilde{X}_T^{(2)}\}\right] \\
&= -\frac{1}{\gamma}e^{\gamma F}\mathbb{E}\Big[\exp\{-\gamma(1-\tau\eta_p)(X_T - X_0)\}\cdot\mathbb{1}_{\{X_T\geq X_0\}} \\
&\qquad\qquad + \exp\{-\gamma(1-\lambda\tau\eta_l)(X_T - X_0)\}\cdot\mathbb{1}_{\{X_T< X_0\}}\Big] \\
&= -\frac{e^{\gamma F}}{\gamma}\Big(\exp\{\gamma(1-\tau\eta_p)X_0\}\cdot g_1(X_0,\tau\eta_p) + \exp\{\gamma(1-\lambda\tau\eta_l)X_0\}\cdot g_2(X_0,\lambda\tau\eta_l)\Big)\,.
\end{aligned}
$$

The first derivative of the expected utility with respect to θ is given by

$$
\frac{\partial EU^{(2)}(\theta)}{\partial\theta} = -\frac{e^{\gamma F}}{\gamma}\left(\exp\left\{\gamma(1-\tau\eta_p)X_0\right\}\frac{\partial g_1(X_0,\tau\eta_p)}{\partial\theta} + \exp\left\{\gamma(1-\lambda\tau\eta_l)X_0\right\}\frac{\partial g_2(X_0,\lambda\tau\eta_l)}{\partial\theta}\right)\,.
$$

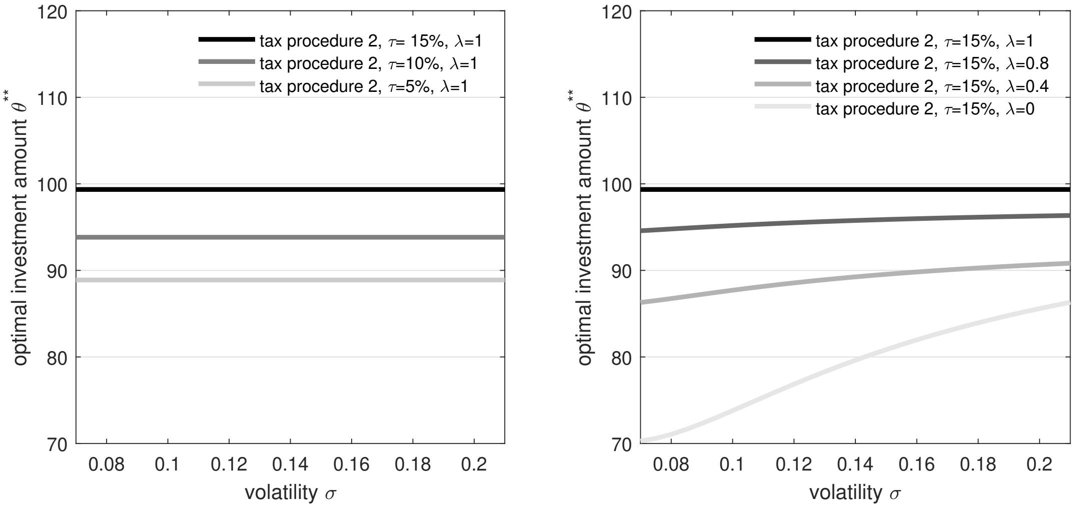
The optimal θ^{**} is either 0, X_0 or determined such that

$$
\frac{\partial EU^{(2)}(\theta)}{\partial\theta} = 0\,. \tag{26}
$$

We can argue similar to the proof of Proposition 2.1 that there exists a unique solution θ^{**} of equation (9).

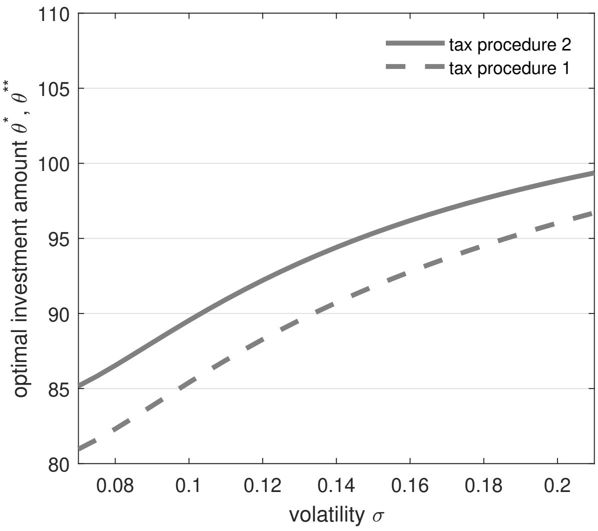
Figures

Figure 1: **Optimal investment amount θ^{**} (with ATR, Procedure 2) for different tax rates and loss offset provisions**



Notes: We choose scenario (1) from Table 1, i.e. $r = 0.02$, $T = 5$, $X_0 = 100$, $\tau = 15\%$, $\delta_p = 4$, $\delta_l = 1$, $d = 0.4$, $F_0 = 0$, $\gamma = 0.02$. The left graph exemplifies symmetric taxation, i.e. taxation with full loss offset ($\eta_p = \eta_l = 1$, $\lambda = 1$). The right graph exemplifies asymmetric taxation, i.e. loss offset restrictions of different levels ($0 \leq \lambda \leq 1$). We choose rate of return on risky investments/risk pairs (μ, σ^2) with identical adjusted Sharpe ratio ASR $= 1.777778$.

Figure 2: **Optimal investment amount θ^* (without ATR, Procedure 1) and θ^{**} (with ATR, Procedure 2)**



Notes: Procedure 1 is indicated by a dashed line (θ^*), whereas Procedure 2 is indicated by a solid line (θ^{**}). We choose scenario (1) from Table 1, i.e. $r = 0.02$, $T = 5$, $X_0 = 100$, $\tau = 15\%$, $\delta_p = 4$, $\delta_l = 1$, $d = 0.4$, $\eta_p = 2.2$, $\eta_l = 1$, $F_0 = 0$, $\lambda = 0.9$, $\gamma = 0.02$, and a rate of return on risky investments and thus risk pairs (μ, σ^2) with identical adjusted Sharpe ratio ASR $= 1.777778$.

Tables

Table 1: **Optimal investment amount and certainty equivalents for Procedures 1 and 2 for slightly restricted loss offset and different levels of risk aversion**

Panel A: Optimal investments with a level of risk aversion of $\gamma = 0.03$					
	$\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l,\ F_0,\ \tau$	θ^*	$CE^{(1)}$	θ^{**}	$CE^{(2)}$
#		$\lambda = 0.9,\ \gamma = 0.03$			
(1)	$(\mathit{4.00}, 1.00, \mathit{0.40}, \mathit{2.20}, 1.00, 0.00, 0.15)$	62.63	111.40	65.06	111.86
(2)	$(\mathit{4.00}, 1.00, \mathit{0.50}, \mathit{2.50}, 1.00, 0.00, 0.15)$	61.91	110.61	64.62	111.08
(3)	$(\mathit{4.00}, 1.00, \mathit{0.60}, \mathit{2.80}, 1.00, 0.00, 0.15)$	61.15	109.84	63.94	110.28
(4)	$(\mathit{3.00}, 1.00, \mathit{0.40}, \mathit{1.80}, 1.00, 0.00, 0.15)$	64.50	112.67	65.35	112.86
(5)	$(\mathit{3.00}, 1.00, \mathit{0.50}, \mathit{2.00}, 1.00, 0.00, 0.15)$	64.32	112.17	65.24	112.36
(6)	$(\mathit{3.00}, 1.00, \mathit{0.60}, \mathit{2.20}, 1.00, 0.00, 0.15)$	64.13	111.67	65.06	111.86

Panel B: Optimal investments with a lower level of risk aversion of $\gamma = 0.2$					
	$\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l,\ F_0,\ \tau$	θ^*	$CE^{(1)}$	θ^{**}	$CE^{(2)}$
#		$\lambda = 0.9,\ \gamma = 0.02$			
(1)	$(\mathit{4.00}, 1.00, \mathit{0.40}, \mathit{2.20}, 1.00, 0.00, 0.15)$	91.80	113.40	95.34	113.93
(2)	$(\mathit{4.00}, 1.00, \mathit{0.50}, \mathit{2.50}, 1.00, 0.00, 0.15)$	90.11	112.39	94.08	112.94
(3)	$(\mathit{4.00}, 1.00, \mathit{0.60}, \mathit{2.80}, 1.00, 0.00, 0.15)$	88.31	111.41	92.41	111.92
(4)	$(\mathit{3.00}, 1.00, \mathit{0.40}, \mathit{1.80}, 1.00, 0.00, 0.15)$	95.26	115.00	96.52	115.22
(5)	$(\mathit{3.00}, 1.00, \mathit{0.50}, \mathit{2.00}, 1.00, 0.00, 0.15)$	94.62	114.35	96.00	114.58
(6)	$(\mathit{3.00}, 1.00, \mathit{0.60}, \mathit{2.20}, 1.00, 0.00, 0.15)$	93.96	113.71	95.34	113.93

Notes: We assume different sets of parameters for our numerical analysis as displayed in rows (1)-(6) including those from our baseline scenario, i.e., a rate of return on the risky investments of $\mu = 0.06$, risk-free rate $r = 0.02$, volatility $\sigma = 0.15$, an investment horizon $T = 5$, initial wealth $X_0 = 100$, tax rate $\tau = 0.15$, a tax loss offset parameter $\lambda = 0.9$ for risk aversion at the level of $\gamma = 0.03$ (Panel A) or a lower level of risk aversion of $\gamma = 0.02$ (Panel B). The profit and loss tax risk multipliers δ_p, δ_l, the probability of a tax reduction after a post-tax audit d are only relevant for the case without ATR (columns θ^* and $CE^{(1)}$) while the profit and loss tax multipliers under ATR-induced tax certainty η_p, η_l and the ATR fee F_0 are exclusively relevant for the case with ATR (columns θ^{**} and $CE^{(2)}$).

Table 2: **Optimal investment amount and certainty equivalents for Procedures 1 and 2 under different loss offset regimes (full or no loss offset)**

Panel A: optimal investments without tax loss offset

	$\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l,\ F_0,\ \tau$	θ^*	$CE^{(1)}$	θ^{**}	$CE^{(2)}$
#		$\lambda = 0,\ \gamma = 0.02$			
(1)	$(\mathit{4.00}, 1.00, \mathit{0.40}, \mathit{2.20}, 1.00, 0.00, 0.15)$	77.83	112.56	80.85	113.01
(2)	$(\mathit{4.00}, 1.00, \mathit{0.50}, \mathit{2.50}, 1.00, 0.00, 0.15)$	76.07	111.61	79.42	112.07
(3)	$(\mathit{4.00}, 1.00, \mathit{0.60}, \mathit{2.80}, 1.00, 0.00, 0.15)$	74.18	110.68	77.62	111.11
(4)	$(\mathit{3.00}, 1.00, \mathit{0.40}, \mathit{1.80}, 1.00, 0.00, 0.15)$	81.18	114.05	82.28	114.24
(5)	$(\mathit{3.00}, 1.00, \mathit{0.50}, \mathit{2.00}, 1.00, 0.00, 0.15)$	80.43	113.43	81.63	113.63
(6)	$(\mathit{3.00}, 1.00, \mathit{0.60}, \mathit{2.20}, 1.00, 0.00, 0.15)$	79.65	112.82	80.85	113.01

Panel B: optimal investments with a full tax loss offset

	$\delta_p,\ \delta_l,\ d,\ \eta_p,\ \eta_l,\ F_0,\ \tau$	θ^*	$CE^{(1)}$	θ^{**}	$CE^{(2)}$
#		$\lambda = 1,\ \gamma = 0.02$			
(1)	$(\mathit{4.00}, 1.00, \mathit{0.40}, \mathit{2.20}, 1.00, 0.00, 0.15)$	93.62	113.51	97.23	114.06
(2)	$(\mathit{4.00}, 1.00, \mathit{0.50}, \mathit{2.50}, 1.00, 0.00, 0.15)$	91.96	112.49	96.00	113.05
(3)	$(\mathit{4.00}, 1.00, \mathit{0.60}, \mathit{2.80}, 1.00, 0.00, 0.15)$	90.17	111.50	94.35	112.03
(4)	$(\mathit{3.00}, 1.00, \mathit{0.40}, \mathit{1.80}, 1.00, 0.00, 0.15)$	97.09	115.12	98.37	115.35
(5)	$(\mathit{3.00}, 1.00, \mathit{0.50}, \mathit{2.00}, 1.00, 0.00, 0.15)$	96.48	114.47	97.87	114.71
(6)	$(\mathit{3.00}, 1.00, \mathit{0.60}, \mathit{2.20}, 1.00, 0.00, 0.15)$	95.83	113.82	97.23	114.06

Notes: We assume different sets of parameters for our numerical analysis as displayed in rows (1)-(6) including those from our baseline scenario, i.e., a rate of return on the risky investments of $\mu = 0.06$, risk-free rate $r = 0.02$, volatility $\sigma = 0.15$, an investment horizon $T = 5$, initial wealth $X_0 = 100$, tax rate $\tau = 0.15$, risk aversion at the level of $\gamma = 0.02$ for either no tax loss offset $\lambda = 0$ (Panel A) or full tax loss offset $\lambda = 1$ (Panel B). The profit and loss tax risk multipliers δ_p, δ_l, the probability of a tax reduction after a post-tax audit d are only relevant for the case without ATR (columns θ^* and $CE^{(1)}$) while the profit and loss tax multipliers under ATR-induced tax certainty η_p and η_l and the ATR fee F_0 are exclusively relevant for the case with ATR (columns θ^{**} and $CE^{(2)}$).

Table 3: **Critical fee levels F_0^* for different levels of risk aversion**

risk aversion	$\gamma = 0.02$	$\gamma = 0.03$	$\gamma = 0.04$	$\gamma = 0.05$	$\gamma = 0.06$	$\gamma = 0.07$	$\gamma = 0.08$
F_0^*	0.13	0.11	0.10	0.10	0.10	0.11	0.11

Notes: We use the set of parameters of the baseline scenario, i.e., a rate of return on the risky investments of $\mu = 0.06$, risk-free rate $r = 0.02$, volatility $\sigma = 0.15$, an investment horizon $T = 5$, initial wealth $X_0 = 100$, tax rate $\tau = 0.15$, for various levels of risk aversion γ. We assume profit and loss tax risk multipliers $\delta_p = 3.0$ and $\delta_l = 0.5$ and a probability of a tax reduction after a post-tax audit of $d = 0.2$ and a tax loss offset parameter $\lambda = 0.5$. We assume profit and loss tax multipliers under ATR-induced tax certainty $\eta_p = 1.40$ and $\eta_l = 0.90$.

Table 4: **Critical fee level F_0^{**} for different tax rate multipliers**

tax rate multiplier	F_0^{**}
$\eta_p = 1.30$, $\eta_l = 1.00$	0.27
$\eta_p = 1.35$, $\eta_l = 0.95$	0.12
$\eta_p = 1.40$, $\eta_l = 0.90$	-0.02
$\eta_p = 1.45$, $\eta_l = 0.85$	-0.16
$\eta_p = 1.50$, $\eta_l = 0.80$	-0.30
$\eta_p = 1.55$, $\eta_l = 0.75$	-0.44
$\eta_p = 1.60$, $\eta_l = 0.70$	-0.58
$\eta_p = 1.65$, $\eta_l = 0.65$	-0.72
$\eta_p = 1.70$, $\eta_l = 0.60$	-0.85

Notes: We use the set of parameters of the baseline scenario, i.e., a rate of return on the risky investment of $\mu = 0.06$, risk-free rate $r = 0.02$, volatility $\sigma = 0.15$, an investment horizon $T = 5$, initial wealth $X_0 = 100$, tax rate $\tau = 0.15$, a tax loss offset parameter $\lambda = 0.5$ and a level of risk aversion $\gamma = 0.04$. We assume profit and loss tax risk multipliers $\delta_p = 3.0$ and $\delta_l = 0.5$ and a probability of a tax reduction after a post-tax audit of $d = 0.2$ and a loss offset parameter $\lambda = 0.5$. We provide critical fee levels F_0^{**} for various sets of profit and loss tax multipliers under ATR-induced tax certainty η_p and η_l.

Table 5: **Definition of variables**

variable	definition
$\{B_t\}_{t\geq 0}$	bond at time $t \in [0, T]$
$CE^{(1)}$	certainty equivalent (no ATR)
$CE^{(2)}$	certainty equivalent (ATR)
d	probability of a tax reduction after a post-tax audit
$ER^{(1)}(\theta)$	expected terminal tax revenue (no ATR)
$ER^{(2)}(\theta)$	expected terminal tax revenue (ATR)
$EU^{(1)}(\theta)$	expected utility (no ATR)
$EU^{(2)}(\theta)$	expected utility (ATR)
$F = F_0 e^{rT}$	accumulated ATR fee
F_0	ATR fee
F_0^*	maximal willingness to pay for an ATR
F_0^{**}	minimal ATR fee
r	risk-free rate
r_τ	interest rate after tax
$\{S_t\}_{t\geq 0}$	risky project at time $t \in [0, T]$
T	investment horizon
$U(X)$	utility function
$\{W_t\}_{t\geq 0}$	Brownian motion at time $t \in [0, T]$
X_0	initial wealth
X_T	terminal wealth of the firm at time T
$\tilde{X}_T^{(1)}$	after-tax net payoff (no ATR)
$\tilde{X}_T^{(2)}$	after-tax net payoff (ATR)
δ_l	tax risk multiplier for losses
δ_p	tax risk multiplier for profits
γ	risk aversion coefficient
η_l	tax rate multiplier for losses under ATR-induced tax certainty
η_p	tax rate multiplier for profits under ATR-induced tax certainty
θ	amount invested in the risky project
λ	tax loss offset parameter
μ	rate of return on risky investments
σ	asset volatility
τ	proportional tax rate for profits and losses
$\tilde{\tau}_l$	stochastic proportional tax rate for losses
$\tilde{\tau}_p$	stochastic proportional tax rate for profits
$\Phi(x)$	standard normal distribution function
$\varphi(x)$	standard normal density function