Faculty of Business Administration and Economics
Chair of Institutional Economics and Economic Policy

Bachelor thesis

Efficiency of Matching Mechanisms –

The Example of Assigning Students to Supervisors

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BSAM –Boston Student Assignment Mechanism

ECTS – European Credit Transfer System

TTC – Top Trading Cycle Mechanism

UPB – University of Paderborn

Introduction

Matching is a process which constantly takes place. For example in everyday life situations, e.g. family members at the breakfast table having different preferences for food, forming groups at the workplace in order to accomplish various tasks, or deciding on which of the friends who are available, one wants to meet after work. Apart from these simple examples, the relevance of matching is considerably higher in other contexts. In the U.S., choosing schools, which is heavily influenced by the results of matching mechanisms, is probably one of the most important decisions of one's life, as it has a severe impact on your future education. Thus, parents start to act strategically at kindergarten level to be able to send their children to their most preferred school.

Since two years, a matching mechanism also affects students at the University of Paderborn (UPB), when assigning them to supervisors for their theses. Having participated in the matching mechanism for central theses application myself, the thereby obtained subjective impression is a major factor of motivation for this thesis. Consequently, it addresses the theory of matching in general and the performance of matching mechanism applied in reality, especially their efficiency with respect to the assignment of students to supervisors at UPB.

To assess matching mechanisms to their full extent, the fundamentals of matching are presented through the example of assigning objects to persons, using bipartite graphs. As the context of UPB requires matching persons to persons, the theory is enhanced to a market in which both sides have preferences. The marriage problem (Gale/Shapely, 1962) introduces all aspects of this two-sided market. Thereafter, the last enhancement of the theory is presented, namely the difference of matching *multiple* students to *one* supervisor. Hence, the college admission problem (ibid), which represents this many-to-one matching, is introduced. Subsequently, the equivalence of the college admission problem to the marriage market and the possibility of preferences being misrepresented is discussed. Besides introducing matching in general, the application of mechanisms applied in reality is addressed as well. This is demonstrated by the context of school choice in which the Boston Student Assignment Mechanism is very common. This mechanism and especially its deficiency that "a student who is not assigned to his top ranked school A is considered for his second choice B only after the students who have top ranked B" (Ergin/Sönmez, 2006, p.216) is discussed in detail. Moreover, the current mechanism at UPB is presented, which is based on the Boston Student Assignment Mechanism. It is discussed whether the current mechanism is efficient for the situation at UPB, despite the mentioned deficiency. The analysis of statistics from former matchings shows that UPB is a competitive environment, in which two alternative mechanisms can be superior in their theoretical properties. Yet it is also established that their instructional capacity and time requirements exceed the possibilities of UPB. All in all, this thesis aims at introducing the matching process in general, as well as discussing whether alternative matching mechanisms are suitable for assigning students to supervisors at UPB.

Matching Markets

As described by Easley and Kleinberg (2010, p.249), the purpose of *Matching Markets* is to match different preferences of individuals for various goods in a way that an efficient outcome is guaranteed. There is a "*bilateral nature* [emphasis added] of exchange" (Roth/Sotomayor, 1992, p.486) in markets where transactions between buyers and sellers of a single commodity take place. These trades are very frequent in reality, which is why it is important to understand that all market participants have objectives for transactions, thus, their preferences, and how to express them, need to be clarified (Shapley/Shubik, 1971, p.111).

Individuals' preferences can be expressed in numerous ways and have been thoroughly analyzed in the field of economics. The characterization of individuals' preferences can be summarized by comparing a student's ranking of schools *x, y*, and *z*. Let the student be called Tom, $x \succcurlyeq y$ denotes that Tom prefers school *x* at least as much as school *y*. When Tom strictly prefers school *x* over school y it is denoted by $x \succ y$. Indifference between the two schools is expressed by $x \sim y$, implying that Tom prefers school *x* at least as much as school *y (x* ≽ y) and he also prefers school *y* at least as much as school *x (y* ≽ *x*) (c.f.Varian,1992,p.94f.). In the same vein, when Tom is not indifferent his preferences are *strict* (Roth/Sotomayor, 1992, p.492f.). To designate the preferences to one individual or a group of agents in a market, for example the preferences of a woman *w*, the preference relation is marked as the following $x \succcurlyeq_w y$ (ibid). When preferences are *complete*, any two schools can be compared, i.e., "*For all* **x** *and* **y** *in X, either* **x** ≽ **y** *or* **y** ≽ **x** *or both*" (Varian, 1992, p.95). In order to guarantee that there is a best school, preferences need to be *transitive*, i.e., "*For all* **x, y** *and* **z** *in X, if* **x** ≽ **y** *and* **y** ≽ **z** *then* **x** ≽ **z**" (ibid).

In addition to this, networks will be used to express preferences and connections. Moreover, complex matching examples will have additional numerical values. To continue, a basic matching task will be introduced, namely an assignment of rooms to people that corresponds to their preferences. The relating framework will be defined hereafter.

Bipartite Graphs

As an introduction, an allocation of goods based on preferences will be illustrated in network form, precisely by means of a *bipartite graph* (see definition below) as illustrated by Easley and Kleinberg (2010, p.250).

To define a bipartite graph, one should note that, in general, graphs are widely used for matching because they can serve as a very comprehensible illustration: A graph shows "a way of specifying relationships among a collection of items. (…) [It] consists of a set of objects, called *nodes*, with certain pairs of these objects connected by links called *edges*" (ibid, p.21).

For defining a bipartite graph, consider the context of two groups of people that are connected with each other, for instance one group called *teachers* and a second group called *students.* The bipartite graph, according to Easley and Kleinberg (2010, p.250), illustrates these two categories as two parallel vertical columns. In these, as mentioned above, teachers and students are represented by two groups of nodes, which are also referred to as *vertices*.[1] In the context of this example, the distinctive feature of a bipartite graph can be explained as the condition that members of the first group are only allowed to lecture members of the second group, hence students never lecture other students and teachers never lecture other teachers (Newman, 2010, p.53).[2] Lecturing is represented as a *link* or edge (see footnote one), illustrated by a line connecting the two columns of nodes. For this reason, an edge links two nodes from opposite categories, i.e., the left or the right column, thus, an assignment is often represented by a link (Easley/Kleinberg, 2010, p.250). An example of a bipartite graph can be seen in *Figure 1.*

In order to present another example of a bipartite matching, bipartite graphs will be defined beforehand. A mathematically precise definition of a bipartite graph by Dhillon (2001, p.269f.) is the following:

> "A graph G = (V, E) is a set of vertices V = {1, 2, . . . |V|} and a set of edges {i, j} each with edge weight E_{ij}. The adjacency matrix M of a graph is defined by
>
> $$M_{ij} = \begin{cases} E_{ij}, & \textit{if there is an edge} \\ 0, & \textit{otherwise} \end{cases}$$
>
> Given a partitioning of the vertex set V into two subsets V_1 and V_2, the cut between them will play an important role (…). Formally,
>
> $$cut\ (V_1, V_2) = \sum_{i \in V_1, j \in V_2} M_{ij} \qquad (1)$$
>
> The definition of cut is easily extended to *k* vertex subsets,
>
> $$cut\ (V_1, V_2, \ldots, V_k) = \textstyle\sum_{i<j} cut\ (V_1, V_2) \quad (2)$$"

A very common bipartite matching assignment will be presented now, followed by assignments that are considered *perfect* and *optimal*. The perfect as well as the optimal assignment will be presented by means of specific examples and will therefore be defined later on. Beforehand, the bipartite matching task, typically an assignment of individuals or objects of one category to

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[1], In the fields of sociology and economics the terms "nodes" and "links" are prevalent, yet in other disciplines different designations, as analogously "vertices" and "edges" occur. Apart from direct quotation, we will mostly use "node" and "link" from here on.

[2] This graph describes an asymmetric relationship, the same is valid for symmetric relationships.

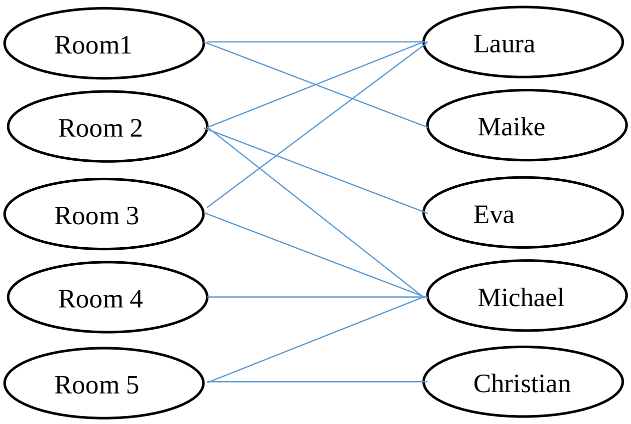
individuals or objects of another category, will be introduced because it is implicit of finding an optimal assignment (Easley/Kleinberg, 2010, p.254.).

Bipartite Matching

In connection to the later addressed issue of assigning students to supervisors at the UPB, the example of a bipartite matching problem will consist of allocating rooms to students living in a shared apartment. Therefore, the two categories that Easley and Kleinberg (2010, p.250ff.) refer to, are students and rooms. And as part of the living agreement the students attempt to find an efficient assignment of rooms to residents.

As can be seen in *Figure 1*, each student is represented by one node on the right, and analogously on the left there exists one node for each room. A link connects a student to a room if he[3] has categorized the room as an acceptable solution and would be willing to move into it (ibid). Thus, Laura's preferences would be met if she lived in the first or the second room whereas Maike would only consider the assignment acceptable if she lived in the first room.

Figure 1: Bipartite Matching: Students and rooms



Source:
Own representation based on Easley/Kleinberz, 2010, p.250

Perfect Matchings

Consequently, according to Easley and Kleinberg (2010), an assignment which guarantees that each resident gives his consent to the living agreement can be described as a *perfect matching*. For the example of *Figure 1* the perfect matching is illustrated in *Figure 3*. One can see that

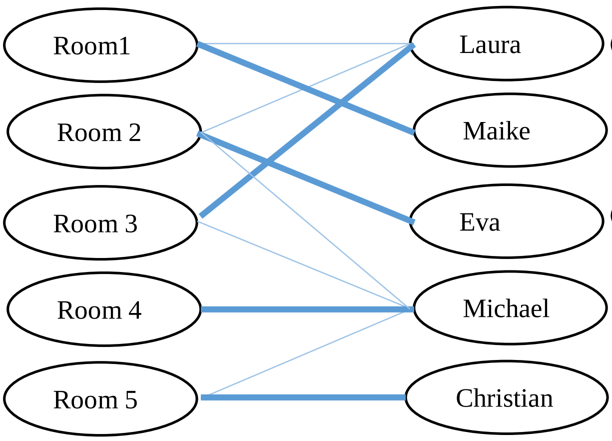
> "*when there are an equal number of nodes on each side of a bipartite graph, a perfect matching is an assignment of nodes on the left to nodes on the right, in such a way that*
> *(i)* *each node is connected by an edge to the node it is assigned to, and*
> (ii) *no two nodes on the left are assigned to the same node on the right*" (ibid).

In anticipation of the assignment of students to supervisors, it is interesting to identify when a bipartite graph does not contain a perfect matching. In that context, it would result in an undesirable situation, (i) when a student *i* is assigned to a supervisor *s* that was not listed as his acceptable option or a situation (ii) in which the number of students assigned to supervisor *s* is higher than the number of students that *s* can supervise. Returning to the example of *Figure 1* and

[3] For ease of exposition, there will not be a gender distinction, instead all individuals (male and female) will only be addressed as "he".

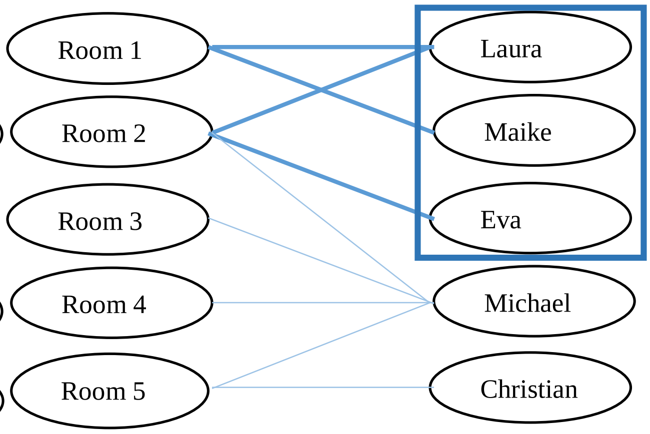
Figure 3, only a slight change regarding the edges, can make a perfect matching impossible. If three people only consider the same two rooms as acceptable, an efficient assignment cannot exist. In *Figure 2,*, the set of Laura, Maike, and Eva has collectively declared just two rooms as acceptable, namely the first and second room. Accordingly, this set is called a *constricted set*, "since their edges to the other side of the bipartite graph 'constrict' the formation of a perfect matching" (Easley/Kleinberg, 2010, p.251). To define this precisely, there is a set of nodes S of a bipartite graph, the residents Laura, Maike and Eva, and a neighbor set of nodes N(S) on the left, the rooms one and two. In N(S), each node has to be linked to at least one node in S. In general, a set is constricted if there are more nodes in S (three in the example) than in N(S) (two in the example) (ibid, p.251f.).

Figure 3: Perfect Matching: Students and rooms



Source:
Own representation based on Easley/Kleinberz, 2010, p.250

Figure 2: Constricted Set: Students and rooms



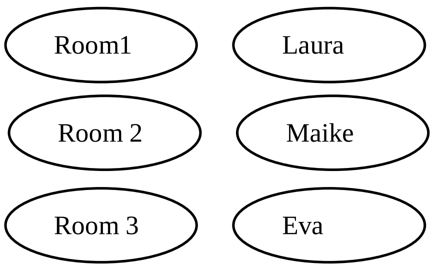
Source:
Own representation based on Easley/Kleinberz, 2010, p.252

Optimal Matchings

Continuing with the example of assigning rooms to students, one can also assume that residents can rank the rooms in addition to only declaring them as acceptable or not acceptable. In that case, as mentioned at the beginning, preferences can be represented by numerical values and each student has a complete and transitive list of preferences.

It is part of the nature of preferences that these rankings are subjective, thus, they can differ individually. One resident can prefer a big room, whereas the other one might choose a smaller room over a bigger one because it is brighter. The residents can rank the rooms according to how much they like each room. Hence, as stated by Easley and Kleinberg (2010, p.253), the preferences can not only be expressed as a binary choice, but also as the residents' *valuations* for the respective rooms. Each resident can rate all rooms; the higher the assigned number, the more they prefer the room. In this example, there are no restrictions or regulations on the scale of the valuations (e.g. total sum for all rooms), instead the residents' valuations are chosen randomly. *Table 1* shows a set of valuations in which all residents prefer the first room, but there are differences in the degree to how much they prefer the first room over their second choice (ibid).

Figure 4: Students' valuations



Source:
Own representation based on Easley/Kleinberz, 2010, p.253

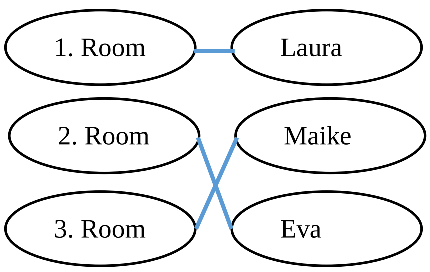
Table 1: Students' valuations

	Room 1	*Room 2*	*Room 3*
Laura	12	2	4
Maike	8	7	6
Eva	7	5	1

Source:
Own representation based on Easley/Kleinberz, 2010, p.253

According to Easley and Kleinberg (2010) an *optimal assignment* for this example, is "the assignment of maximum possible quality, (…) [maximizing] the total happiness of everyone for what they get" (ibid, p.254). Naturally only one resident can live in the first room, so maximizing the quality of an assignment, and therefore the common good, does not imply the maximization of each individual's welfare. *Figure 5* shows the optimal assignment for this set of valuations. The explanation why this is optimal, in terms of the definition above, will be outlined briefly.

Figure 5: Optimal Matching: Students and rooms



Source: Own representation based on Easley/Kleinberg, 2010, p.253

Table 2: Optimal matching

	Room 1	*Room 2*	*Room 3*
Laura	**12**	2	4
Maike	8	7	**6**
Eva	7	**5**	1
Result:		**23**	

Source:
Own representation

The sum of the valuations[4] of the assigned room provides a score that facilitates to evaluate the efficiency of the assignment. In this case the optimal assignment results in 23 units (see *Table 2*), thus 23 can be considered the *social value* of the assignment, representing how much society, in this case the community of residents, benefits from this specific distribution of rooms (c.f. ibid). The difference between Laura's evaluations is very drastic. If the second or third room had been assigned to her, Laura's contribution to the social value would have been a lot smaller (see *Table 3*).

[4] It is referred to "*units*" of valuations from now on as well.

Table 3: Optimal Matching: Laura's options

	Room 1	Room 2	Room 3
Laura	12	2	**4**
Maike	**8**	7	6
Eva	7	**5**	1
Result:		17	

	Room 1	Room 2	Room 3
Laura	12	**2**	4
Maike	8	7	**6**
Eva	**7**	5	1
Result:		15	

Source: Own representation

Having assigned Laura to the first room, the difference of Maike's options are analyzed. She has evaluated the difference between room three and room two by only one unit, so either way she would contribute roughly the same. Now considering Eva's evaluation for the second and third room, *Table 4* clearly shows that when Eva is assigned to the third room it results in a lower social value. This cannot be compensated by the additional unit that Maike then contributes.

Table 4: Optimal Matching: Eva's and Maike's options

	Room 1	Room 2	Room 3
Laura	**12**	2	4
Maike	8	7	**6**
Eva	7	**5**	1
Result:		23	

	Room 1	Room 2	Room 3
Laura	**12**	2	4
Maike	8	**7**	6
Eva	7	5	**1**
Result:		20	

Source: Own representation

One should note here that finding an optimal assignment resolves the bipartite matching problem as well, because the binary choice between acceptable or not acceptable can be expressed by numerations of one and zero (Easley/Kleinberg, 2010, p.254). An important insight for the further proceeding is that, whenever individuals evaluate a collection of objects, the quality of an assignment of these can as well be evaluated as a measurement for social welfare (ibid, p.253).

Two-Sided Matching

The introduction of valuations is beginning to advance the model with respect to the main task – assigning students to supervisors. For further progress, the focus will now turn to the extension that, not only students have rankings over their desired supervisor, but also supervisors do have preferences on the students they work with, as for example their course choices or grades. Therefore, there is a "bilateral nature of exchange" (Roth/Sotomayor, 1992, p.486).
Thus, an important addition to the theory introduced so far will now be the fact that the market is *two-sided*. Roth and Sotomayor (1992) define the phrase *two-sided matching markets* as "the fact that agents in such markets belong, from the outset, to one of two disjoint" (ibid) categories – e.g. students and supervisors. David Gale and Lloyd Shapley introduced such a two-sided matching

model in their paper *College Admissions and the Stability of Marriage* (1962) with an assignment problem known as the *marriage model*.

The Marriage Model - One-to-One Matching:

Prior to the presentation of a *stable* solution for the marriage problem, which cannot be upset by any agents "acting together in a manner which benefits both [agents]" (Gale/Shapley, 1962, p.10), the framework, according to Roth and Sotomayor (1992, p.492f.), is now outlined: "Two finite and disjoint set of agents" (ibid, p.492), which can be illustrated by a bipartite graph, are called the set of men M=$\{m_1, m_2, \dots, m_n\}$ and women W=$\{w_1, w_2, \dots, w_n\}$. Each agent has transitive and complete preferences over the other set of agents. The preferences of each woman is represented by an ordered list P(w), on the set $M \cup \{w\}$, for example P(w)= $m_2, m_1, w, m_3, \dots, m_n$ which is abbreviated by ending at the woman's preference of remaining single. This is represented by w, in this case the woman's third priority.[5] A marriage market consists of M, W, and **P**, in which **P** represents the set of preference lists of all men and women. The general rules of matching are that if there is mutual consent about the matching of any agent to another agent from the opposite category, they may be matched. An outside option is always to remain unmatched, if any agent prefers to do so. A matching μ is called *individually rational* if no agent x favours being unmarried to $\mu(x)$ (Roth, 1985, p.278). Hence, a "*matching μ* is a one-to-one correspondence from the set $M \cup W$ onto itself of order two (…) such that if $\mu(m) \neq m$, then $\mu(m)$ is in W and if $\mu(w) \neq w$, then $\mu(w)$ is in M. We refer to $\mu(x)$ as the mate of x" (Roth/Sotomayor, 1992, p.493). [6]

As mentioned previously, this matching can be illustrated by a bipartite graph, possessing the main characteristic that "the edges in a bipartite network run only between vertices of unlike types" (Newman, 2010, p.123). Accordingly this definition of matching not only implies that agents are only matched to agents from the opposite set, but also that the matching is *bilateral* i.e. that when "man *m* is matched to woman *w*, then woman *w* is matched to man *m*" (Roth/Sotomayor, 1992, p.493).

Additionally, Roth and Sotomayor assume that the preference for the matching corresponds to the preference for the matched agent, therefore, m prefers matching μ to matching v under the condition that he prefers $\mu(m)$ to $v(m)$. Later this will be more important when continuing with the assignment of a multitude of agents to one institution. For now, each agent from one category can be matched to no more than one agent from the other category. Thus, it is referred to as *one-*

[5] Analogously for the set of men: P(m), on the set $W \cup \{m\}$, for example P(m)= $w_2, w_1, m, w_3, \dots, w_n$, implying that *m* represents the man's preference of remaining unmarried.
[6] We are thus only discussing heterosexual couples here.

to-one matching (ibid, p.491ff.). As in the example of assigning rooms to students, only one man can be married, i.e., assigned to one woman, but in contrast to the first example, both agents have preferences.

Stable Matching

Each agent is assumed to act in a way that satisfies his preferences best, i.e., maximizes his utility, consequently each agent will try to improve his situation by "acting together" (Gale/Shapley, 1962, p.10) with other agents so that it is profitable for both agents. Hence, Gale and Shapley (1962) strive for a resistant way of marrying off all members of a community that consists of *n* men and *n* women, they state that therefore the set of marriage must be *stable*. A set of marriage is defined as *unstable*, "if under it there are a man and a woman who are not married to each other but prefer each other to their actual mates" (ibid, p.11). In the same vein, this attribute which in other contexts is referred to as *justified envy* (Abdulkadiroğlu/Sönmez, 2003, p.731) leads to market failure, thus, for most assignments it is necessary that matchings "cannot be improved upon by any individual or any pair of agents" (Roth/Sotomayor, 1992, p.494).

In the following example of the marriage market, each agent from the two sets $M = \{m_1, \ldots, m_n\}$ called men and $W = \{w_1, \ldots, w_n\}$ called women, has a complete and transitive preference ordering over the agents from the opposite set as well as to remain unmatched and stay single (Roth/Vande Vate, 1990, p.1475). In order to find a stable set of marriages Gale and Shapley (1962, p.11) use *ranking matrices* as portrayed in *Table 5*. In this example, Patrick rates Laura as the most desirable option (one), then Maike (two), followed by Eva (three). Reversely Patrick is ranked number three by Laura, as number two by Maike and as number one by Eva. Hence, the first number of each pair in the matrix represents the men's ranking of the women and the second number illustrates how the women rank the men. In this example six sets of marriages are possible,[7] yet only half of them are stable matchings.

Table 5: Marriage Market

	Laura	*Maike*	*Eva*
Patrick	1, 3	2,2	3,1
Christian	3,1	1,3	2,2
Andreas	2,2	3,1	1,3

Source:
Own representation based on Gale/Shapley, 1962, p.11

[7] "The number of possible matchings equals $n!$, the number of permutations of n elements" (Maschler et al., 2013, p.892).

According to the definition above, one can identify the stable sets by verifying that none of the matched couples would split up because no agent *a* is able to find a more preferred partner that favours agent *a* to his current mate. One stable set in this example is assigning all women to their most preferred man (Laura & Christian, Maike & Andreas, Eva & Patrick). The specific feature here is that this set is stable although each man is assigned to his least preferred woman. In *Table 6, 7,* and *8* matchings of this set are illustrated. The initial situation is indicated in bold and the suggested change is indicated in italics. An arrow pointing to the left represents an improvement, as the tables show preference *rankings*. To proof stability for this specific set one can choose two couples, e.g. Maike & Andreas and Laura & Christian. The men are willing to switch partners (see *Table 7*), but as illustrated in *Table 6* the new situation would be less desirable for the women, so Maike would improve her new situation by "acting together" (ibid, p.10) with Patrick as they both prefer each other to their mates at that point (see *Table 8*). Therefore, the new situation is unstable.

Table 7: Marriage Market: Men's options

P(Andreas)	Eva	*Laura*	**Maike**
P(Christian)	*Maike*	Eva	**Laura**

Source: Own representation

Table 6: Marriage Market: Women's options

P(Maike)	**Andreas**	Patrick	*Christian*
P(Laura)	**Christian**	*Andreas*	Patrick

Source: Own representation

Table 8: Marriage Market: Coalition

P(Maike)	Andreas	*Patrick*	***Christian***
P(Patrick)	Laura	*Maike*	**Eva**

Source: Own representation

It can easily be verified, that another stable arrangement in this example is to pair all women and men who rated each other as a two (Laura & Andreas, Maike & Patrick, Eva & Christian) and that the third stable set is when all men's preferences are met perfectly (Patrick & Laura, Christian & Maike, Andreas & Eva) (ibid, p.11).

In order to find an efficient way of determining stable matchings, an algorithm that was published by Gale and Shapley in *The American Mathematical Monthly* in 1962 will now be introduced. Yet, it should be noted that an almost equivalent algorithm was already in use for the *National Intern Matching Program* in the United States since 1951 to assign internship positions at hospitals to newly-graduated medical students (Roth/Sotomayor, 1992, p.486ff.). However, the case of the matching market for American physicians will be approached after having explained the algorithm as it is an example of many-to-one matching which will be discussed in detail at a later point.

Deferred Acceptance Mechanism

Gale and Shapley developed a theorem that “there always exists a stable set of marriages” (1962, p.12) when an iterative procedure is applied. This “*deferred acceptance procedure* [emphasis added]” (ibid, p.13) that results in a stable matching for the marriage market can be categorized as an algorithm applied from either the women’s perspective, then named the “*women’s courtship algorithm*” (Maschler et al., 2013, p.891) or the men’s perspective (“*men’s courtship algorithm*” (ibid, p.890)). The latter, which is analogous to the first, is described as the following by Gale and Shapley (1962, p.12f.). Each man suggests an engagement to his most preferred woman. It can occur that some women now have multiple or no offers to choose from, each woman then chooses her favourite man out of her admirers and they get engaged. The men who were not chosen by their first choice will now suggest an engagement to their second most preferred woman. Now, the women again have the option to stay single, choose a new man or remain engaged, i.e., if a woman is engaged but prefers the new proposer to her fiancé, she would break off the engagement and accept the new proposal. This will continue until all rejected men had the possibility to propose to all women, who always reject all but their most preferred man that has proposed so far. The procedure will not exceed $n^2 - n + 2$ stages because a man cannot offer an engagement to a woman more than once. When the last woman chooses to get/remain engaged or stay single, this set of couples is considered stable, so the time of *courtship* is over and the couples will get married (ibid, p.12f.).

Applying the men’s courtship algorithm to the example of *Table 5*, it leads to the matchings of Patrick & Laura, Christian & Maike, Andreas & Eva, which as mentioned, is the best stable matching for the men. Consulting the ranking matrix again, it also shows that it is the worst outcome for the women. In this case, applying the women’s courtship algorithm results in the matchings Laura & Christian, Maike & Andreas, Eva & Patrick. Reversely, this is the best outcome for the women and the worst for the men. Maschler et al. (2013, p.894f.) prove that this is the case for every stable matching. Thus it is valid that for every stable matching μ, the *men’s courtship matching* O^m and the *women’s courtship matching* O^w, “one has $O^m \succcurlyeq_m \mu \succcurlyeq_m O^w$ and $O^w \succcurlyeq_w \mu \succcurlyeq_w O^m$” (ibid, p.894).

Before continuing with the matching market for American physicians, one should note that, as shown in *Table 6, 7* and *8*, despite the declared ambivalent outcome, the deferred acceptance mechanism still leads to a stable matching. In conclusion, Gale and Shapley (1962) require an assignment to be stable; this is also adequate for the assignment of students to supervisors at UPB. A situation where a mechanism matches two students to two supervisors and both students prefer the other student’s supervisor over their own, would lead to great dissatisfaction amongst students and therefore complications for the university. Having introduced O^w and O^m, the question emerges whether there also exists one stable outcome that is preferred by all students or/and all supervisors. Therefore, it is referred back to the *National Intern Matching Program* and the context of students, more precisely to an issue named *college admission problem*. Later the options of unstable but Pareto dominating matchings will be analyzed as well.

College Admission Problem – Many-to-One Matching

As mentioned earlier, a deferred acceptance mechanism was already in use in a market categorized by the set of hospitals on the one side and the set of graduating medical students on the other side. In this labor market each student wants to be matched to one job and each hospital wants to employ a certain number of students as resident physicians. Therefore, it is analogous to the *college admission problem* (Roth, 1985, p.280). Also in this problem, one of the two disjoint sets is represented by one single institution and the other one by many students. Hence, we are now looking at *many-to-one matching*, instead of one-to-one matching, to which the example of assigning students to supervisors at UPB belongs as well. This extension needs to be reviewed to see whether results and definitions from simple one-to-one matchings are valid for many-to-one matchings (Roth, 1985, p.277).

Equivalence to the Marriage Market

The general context from now on will be two finite and disjoint sets of colleges and students, $\mathcal{C} = \{C_1, C_2, \dots, C_n\}$ and $\mathcal{S} = \{s_1, s_2, \dots, s_m\}$ respectively, in which each college has strict preferences over each student and each student equivalently has strict preferences over each college. Given that the *quota* q_c represents how many students a college accommodates, all q_c positions are regarded as equal because, as Roth and Sotomayor (1992) note, students' preferences are over colleges in general and not over being one of the colleges' top candidates (ibid, p.494f.). In this context, a matching of students to colleges is bilateral. When man *m* is matched to woman *w* in the marriage market, then woman *w* is matched to man *m*, analogously in the college admission problem "a student is enrolled in a college if and only if the college enrolls that student" (ibid). In the same vein, a solution to this matching assignment must be in compliance with the conditions "that each student is matched to at most one college, and each college is matched to at most its quota of students". Moreover the possibilities of agents being "matched to themselves" (ibid), i.e., not enrolling at any college or not achieving a college's quota, must be considered as well.

According to what was just stated, the formal definition of

> "a matching μ is a function from the set $\mathcal{C} \cup \mathcal{S}$ into the set of unordered families of elements of such that:
> (1) $|\mu(s)| = 1$ for every student s and $\mu(s) = s$ if $\mu(s) \notin \mathcal{C}$;
> (2) $|\mu(\mathcal{C})| = q_c$ for every college *C*, and if the number of students in $\mu(C)$, say *r*, is less than q_c, then in $\mu(C)$ contains $q_c - r$ copies of *C;*
> (3) $\mu(s) = C$ if and only if s is in $\mu(C)$" (ibid)

Roth (1985, p.281) argues that such a formulation of the college admission problem is not complete enough because a college's preferences over matchings are not specified, only the preferences for individual students. Every college, which enrolls more than one student, can only compare its options when it can compare groups of students. Therefore, preferences of colleges over groups must be specified as well. This is what differentiates one-to-one from one-to-many matching. Hence, we now define "the *preference relation $P^{\#}(C)$* of college *C* over all assignments $\mu(C)$ it could receive at some matching μ*"* (Roth/Sotomayor, 1992, p.496). In order to analyze the equivalence of the college admission problem to the marriage market, an important aspect of a college's preference relation over *sets of* students, is its deviation from the college's preferences over *individual* students, thus this will be characterized now (Roth/Sotomayor, 1992, p.495f).

Responsive preferences and stable matchings

As stated before, the preferences over individual students are denoted as an ordered list *P(C)*. A preference relation *$P^{\#}(C)$* over sets of students from college *C* with $q_c = 2$ can be exemplified as $P\#(C) = \{s_1, s_2\}, \{s_1, s_3\}, \{s_2, s_3\}, \{s_3\}, \{s_2\}, \{s_1\}$. This preference relation shows that the college individually prefers s_3 to s_1 or s_2, but concerning pairs of students, $\{s_1, s_2\}$ is favoured to $\{s_1, s_3\}$ this preference relation is therefore not *responsive* (Roth/Sotomayor, 1992, p.502). The preference relation $P^{\#}$(C) over groups is responsive to its preferences P(C) over individuals "if, for any two assignments that differ in only one student, it [i.e. college C] prefers the assignment containing the more preferred student" (ibid, p.496).[8] Roth and Sotomayor (ibid, p.497) stress that under the condition of preferences being responsive, the definition of a stable matching by Gale and Shapley is also adequate for many-to-one matching (ibid).[9] Hence, one could conclude that responsiveness prevents having to define complete preference relations of all supervisors at UPB.

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[8] A detailed definition of responsiveness, by Roth and Sotomayor, is that the "preference relation $P^{\#}$(C) over sets of students is responsive [to the preferences P(C) over individual students] if, whenever $\mu'(C) = \mu(C) \cup \{s_k\} \setminus \{\sigma\}$ for σ in μ(C) and s_k not in μ(C), then C prefers μ'(C) to μ(C) [under $P^{\#}$(C)] if and only if C prefers $\{s_k\}$to σ [under P(C)]" (1992, p.496).

[9] A counter example for a stable matching, i.e., an unstable matching in this context is when "the pair (*C*, *s)* can improve upon μ if $\mu(s) \neq_F C$ and if $C \succ_s \mu(s)$ and $s \succ_c \sigma$ for some σ in μ(C). [Note that σ may equal either some student *s'* in *μ(C)*, or, if one or more of college C's positions is unfilled at *μ(C), σ* may equal C]" (Roth/Sotomayor, 1992, p.497). Roth and Sotomayor (1992, p.494) refer to individuals or pairs dominating unstable matchings as *coalitions. I*f preferences were not responsive one would have to consider "coalitions consisting of colleges and several students (…), or even coalitions consisting of multiple colleges and students" (ibid, p.497).

Nevertheless it should already be noted that the preferences of supervisors are considered to be determined exogenously and are thus a special case. This will be further defined in the context of school choice in the next section. Before, out of reasons of formal completeness, it will be examined whether the statement by Gale and Shapley (1962, p.12), that the deferred acceptance mechanism *always* results in a stable matching, is also adequate for the college admission problem.

Substitutable preferences and the nonempty set of stable matchings

Roth and Sotomayor clarify that, also for the earlier mentioned theorem, that "the set of stable matchings is always nonempty" (Gale/Shapely, 1992, p.497ff.), an important prerequisite exists for many-to-one matchings. They consider the context of matching employees to firms and state that under the condition that the firms' preferences are *substitutable*, the theorem is true. This explanation by Roth and Sotomayor will be applied onto the context of assigning students to supervisors to briefly define substitutability. Given that preferences are substitutable, a chair does not regard students as complements. If two students apply to write their thesis at chair *h*, one student who is very good in presenting essays (but only that) and one who is very good at only writing essays, chair *h* will accept both, independent from the question if only one or both students will end up writing their thesis with them. It could be assumed that if only one student enrolls, chair *h* would prefer a different student altogether, namely someone who is able to present *and* write on an average level. So given that preferences are substitutable a situation, in which a chair only accepts the two students if both write their thesis at that chair, will not occur (ibid, p.499).

Having shown that in the college admission problem, under certain conditions, the deferred acceptance algorithm will result in a stable matching that always exists, the question whether there also exists one stable outcome that is preferred by all students or/and all supervisors can finally be addressed. Before this matching, in which "every applicant is at least as well off (…) as he would be under any other stable assignment" (Gale/Shapley, 1962, p.14) is found, the so far discussed prerequisites by Roth and Sotomayor (1992) for the equivalence of the college admission problem to the marriage market are summarized in *Table 9.*[10]

[10] O^m is the men's courtship matching and O^w the women's courtship matching, as defined on page eight. O^c is the college optimal stable outcome, the result of the deferred acceptance mechanism applied from the perspective of the colleges, and in the same vein O^s is the student optimal stable outcome.

Table 9: Equivalence of the Marriage Market and the College Admission Problem I

Deferred Acceptance Mechanism always results in				
Context	Marriage Market		College Admission Problem	
***Always** existing matching*	O^m	O^w	O^c, if colleges' preferences are substitutable	O^s, if colleges' preferences are substitutable
Property	stable	stable	stable, if colleges' preferences are responsive	stable, if colleges' preferences are responsive

Own representation

Optimality

It was already mentioned that the deferred acceptance mechanism, resulting in O^m or in O^w, provides the best outcome for men or women respectively. This will initially be analyzed for the college admission problem, as the context of UPB is a special case that will be addressed in the next section. For the context of colleges and students, the algorithm proceeds exactly like in the marriage market as presented in detail earlier. The second theorem by Gale and Shapley in their paper *College Admission and Stability of Marriage* from 1962 states that "every applicant is at least as well off under the assignment given by the deferred acceptance procedure as he would be under any other stable assignment" (p.14), thus, the resulting assignment is called *optimal* (ibid, p.10f.). Hence, in addition to the property of stability for O^c and O^s, which was proven in the last section, Gale and Shapley assume that the deferred acceptance mechanism also leads to the above defined optimal assignment as the "procedure only rejects applicants from colleges which they could not possibly be admitted to in any stable assignment" (ibid, p.14).

So one can assume that the optimal assignment, derived from the deferred acceptance mechanism is at least as preferred as *all other stable* matchings. In addition to that, Roth (1985, p.280) specifies for the context of the marriage market, when comparing O^m and O^w with *all possible* matchings, for no matching *y*, which could also be an *unstable* assignment, is it the case that $y \succ O^m$ for all *m* in *M*. Equivalently, no matching *z* exist which is preferred by all the women to O^w. Hence, for the marriage problem it is valid that O^m/O^w is weakly Pareto optimal for either category of agents respectively (Roth/Sotomayor, 1992, p.507).

Yet, Roth (1985) argues that in the context of the college admission problem there does exist an unstable outcome that all colleges strictly prefer to the college-optimal stable outcome O^c, when they have responsive preferences. This is verified by the following example which is derived from Roth (1985, p.283). Consider that the preferences are responsive and substitutable. Furthermore, as in the marriage market P(c) = c and P(s) = s represent the preference to stay unmatched.

The preference orderings of three colleges $C = \{c_1, c_2, c_3\}$ are • P(c_1)= $s_1, s_2, s_3, s_{4,}c$, • P(c_2)= $s_1, s_2, s_3, s_{4,}c$, and • P(c_3)= $s_3, s_1, s_2, s_{4,}c$. The colleges' quotas are $q_1 = 2,\ q_2 = 1,\ q_3 = 1$.	Four students $S = \{s_1, s_2, s_3, s_4\}$ have the preference orderings • P(s_1)= c_3, c_1, c_2, s , • P(s_2)= c_2, c_1, c_3, s, • P(s_3)= c_1, c_3, c_2, s, and • P(s_4)= c_1, c_2, c_3, s
Following the deferred acceptance mechanism, the resulting college optimal stable matching O^c is • $O^c(c_1) = \{s_3, s_4\}$, • $O^c(c_2) = \{s_2\}$, • $O^c(c_3) = \{s_1\}$.	Roth provides the outcome *y*: • $y(c_1) = \{s_2, s_4\}$, • $y(c_2) = \{s_1\}$, • $y(c_3) = \{s_3\}$,

Matching *y* is represented by the italic, blue-highlighted characters in *Table 10* and $O^c$is indicated in bold. As illustrated by the arrows, the unstable outcome *y* Pareto dominates the stable outcome O^c from the perspective of the colleges, as all colleges strictly prefer it.

Table 10: Comparison of the college optimal stable matching O^c and the Pareto dominating matching y

P(c1)	S1	*S2*	**S3**	***S4***
P(c2)	*S1*	**S2**	S3	S4
P(c3)	*S3*	**S1**	S2	S4

Source: Own representation

This example has shown that "there is a potential conflict between complete elimination of justified envy [, i.e., stability] and Pareto efficiency" (Abdulkadiroğlu/ Sönmez, 2003, p.732). This conflict exists also for students but to a different degree. There is no outcome *y* which is strictly preferred by all students (Roth, 1984, p.287) but Abdulkadiroğlu and Sönmez (2003, p.736) present an example in which out of three students, two strictly prefer an unstable outcome *y* to O^s and one student is indifferent between O^s and *y*. Hence, in the aspect of Pareto efficiency of the deferred acceptance mechanism result, the college admission problem is not equivalent to the marriage market (Roth/Sotomayor, 1992, p.507). This means that also at UPB one must prioritize either Pareto efficiency or stability. The resulting complications will be discussed further when comparing outcomes of alternative mechanisms.

An important new aspect of matching markets that will be discussed from here on is that each agent always wants to maximize his own benefits. Therefore, he can also consider a strategic misrepresentation of his preferences if that results in a more efficient outcome. For a marriage market with strict preferences it is known, that when any stable mechanism is applied, and there is more than one stable matching, then at least one agent can profitably misrepresent his preferences, if the others tell the truth. A more detailed analysis follows (ibid, p.517).

Truthful preference revelation

Having identified outcomes that are preferred to the optimal stable outcome, Roth (1985, p.285ff.) emphasizes an aspect that is a major issue in reality, e.g. at UPB, which is agents misstating their preferences. Roth concludes that for the college admission problem, and therefore also for the marriage market,[11] no stable matching procedure induces *all* agents to always state their true preferences. Nevertheless we have seen examples of the marriage market in which *one category of agents* obtained their most preferred assignment by stating their true preferences: the women's courtship algorithm and the men's courtship algorithm. This entails that for one-to-one matching, it is valid that women's courtship algorithm makes it a dominant strategy for all women to state their true preferences and equivalently the men's courtship algorithm makes it a dominant strategy for all men to state their true preferences (ibid, p.280). Therefore, the mechanism is *strategy proof "*(, i.e., it cannot be manipulated by [an individual who is] misrepresenting [his] preferences)" (Abdulkadiroğlu/Sönmez, 2003, p.731) for respectively one category of agents.

Roth (1985, p.285ff.) again proves that this is not completely the case for the college admission problem when colleges have responsive preferences. The example that proved the existence of a matching that Pareto dominates O^c also implies that there does not exist a strategy so that every college always states its true preferences. *Table 10* shows that under the deferred acceptance mechanism c_1 is matched to its third and fourth preference. If c_1 wants to improve the outcome for itself, it would state P'(c_1)= s_2, s_4, c, s_1, s_3 instead of its true preferences P(c_1)= s_1, s_2, s_3, s_4, c. Given that all other colleges state their true preferences, a new stable outcome occurs. It is the already discussed Pareto dominating matching y, indicated in italic, blue-highlighted characters in *Table 10,* with $y(c_1) = \{s_2, s_4\}, y(c_2) = \{s_1\}$, and $y(c_3) = \{s_3\}$. Due to that, an incentive to misrepresent preferences is created because c_1 will maximize its benefits by doing so. *Table 11*

[11] Consider the marriage market as a special case of the college admission problem in which all colleges have the quota one: q_i=1.

again shows all discussed differences for the college admission problem when compared to one-to-one matching (Roth, 1985).

Table 11: Equivalence of the Marriage Market and the College Admission Problem II

Deferred Acceptance Mechanism always results in

	Marriage Market		College Admission Problem	
Matching	O^m	O^w	O^c, if colleges' preferences are substitutable	O^s, if colleges' preferences are substitutable
Property	stable	stable	stable, if colleges' preferences are responsive	stable, if colleges' preferences are responsive
	$\geq_m$ all other *stable* matchings	$\geq_w$ all other *stable* matchings	$\geq_c$ all other *stable* matchings	$\geq_s$ all other *stable* matchings
	$>_m$ any *possible* matching (Pareto optimal for M)	$>_w$ any *possible* matching (Pareto optimal for W)	An outcome exists that all colleges strictly prefer to O^c.	No outcome exists that *all* students *strictly* prefer to O^s.
	Mechanism that leads to O^m: All men state their true preferences	Mechanism that leads to O^w: All women state their true preferences	No stable matching procedure makes it a dominant strategy for every *c* to state its true preferences	Mechanism that leads to O^s: All students state their true preferences

Source: Own representation

The conclusion that someone can "profitably misrepresent [his/] her preferences" (Roth/Sotomayor, 1992, p.518) indicates that it is necessary to consider consequences of matching mechanisms applied in reality. An extensively researched field, where matching mechanisms are applied, is the education sector. Especially the *Boston Public School System* for assigning students to schools has been analyzed, improved and adapted by various researchers and institutions (Abdulkadiroğlu et al., 2005b, p.368). The procedure for the assignment of students to supervisors at UPB is also based on the so called *Boston Student Assignment Mechanism*. Hence, this mechanism will now be introduced in the context of *school choice* (see definitions below).

School choice – The Boston Student Assignment Mechanism

School choice means parents have "the opportunity to choose the school their child will attend" (Abdulkadiroğlu/Sönmez, 2003, p.729). Apart from the outside option of a private school, parents had to accept the school which was assigned to their children by the district they live in, before *intra-district* and *inter-district* choice programs were available.[12] Now that parents have multiple

[12] "Intra-district choice allows parents to select schools throughout the district where they live, and inter-district choice allows them to send their children to public schools in areas outside their resident districts" (Abdulkadiroğlu/Sönmez, 2003, p.729).

options, each student[13] has strict preferences over all schools and although there exist sufficient seats for students in total, each school has limitations on the maximum number of seats (ibid, p.733). Michelle A. Hernández, former assistant director of admissions at Dartmouth College, who is now working as a very exclusive college admissions coach, points out the 2011 admission rates of Ivy League universities on her official website. As can be seen in *Table 12* applicants face intense competition.

Table 12: Competition level for Ivy League Universities

Ivy League	*Admitted*	*Applied*	*Acceptance rate*
Harvard University	2,058	22,955	8,97%
Princeton University	1,791	18,942	9,46%
Yale University	1,860	19,323	9,63%
Columbia University	2,255	21,343	10,57%
Brown University	2,683	19,097	14,05%
Dartmouth College	2,166	14,176	15,28%
University of Pennsylvania	3,637	22,646	16,06%

Source: Own representation based on Hernández College Consulting: Ivey League Admission Statistics for class of 2011, 2014

Due to this, Hernández notes in her book "*A Is for Admission: The Insider's Guide to Getting into the Ivy League and other Top Colleges*" (1997), that parents should already be aware of the differences between schools that their child can attend. Especially in high schools, not only the quality of the college counselor varies, but also the preparations for SAT and application letters differ from school to school. Hence, choosing a high school is of great importance for the future education of a child. Hernandez emphasizes that admission officers are assigned regions for which they become experts, so popular colleges collect information on high schools e.g. their "grading systems, representative course loads, where [their] students typically attend college" (Hernandez, 1997, p.13) and in the case that former students of this high school attend their college also the students' GPA.

Having illuminated the background of school choice and its significance, the design of a *student assignment mechanism* (Abdulkadiroğlu/Sönmez, 2003, p.729) will be discussed from here on. This can be regarded as almost equivalent to the college admission problem as it "is a systematic procedure that selects a matching for each school choice problem" (ibid, p.733) so that every student is matched to one school and no school must enrol more students than it has seats for. In school choice the essential difference is, that schools are only objects to be consumed by the students, while in the college admission problem schools have their own preferences over students (ibid, p.731). Hence, in the college admission problem "both sides of the market are strategic

[13] Although in reality it is rather the choice of the parents, we will refer to this category of agents as *students*.

actors" (Chen/Sönmez, 2006, p.203) whereas in school choice only the welfare of the student matters (Pathak/Sönmez, 2008, p.1639). Nevertheless, schools have *priorities* instead of preferences, these *priority orderings* are based on state and local laws,

> *"for example:*
> - *students who live in the attendance area of a school must be given priority for that school over students who do not live in the school's attendance area,*
> - *siblings of students already attending a school must be given priority, and*
> - *students requiring a bilingual program must be given priority in schools that offer such programs"* (Abdulkadiroğlu/Sönmez, 2003, p.731).

If two students are identical in all of the above listed priorities, a lottery will decide which student is to be preferred (ibid, p.733). This exogenous determination of priorities results in the above stated fact that only actors on one side of the market are strategic, namely the students (Chen/Sönmez, 2006, p.203). On this basis, school choice programs, as applied for example in Boston, Minneapolis, and Seattle, have severe deficiencies. "Under these procedures students with high priorities at specific schools lose their priorities unless they list the schools as their top choices" (Abdulkadiroğlu/Sönmez, 2003, p.730) as "a student who fails to get her first choice may find her later choices filled by students who chose them first" (Abdulkadiroğlu et al., 2005b, p.368). Later, this will be discussed in detail. Despite this deficiency, a very common mechanism in practice is the *Boston Student Assignment Mechanism (BSAM)* named so as the city of Boston has used this mechanism for their school choice problem since July 1999 (Abdulkadiroğlu/Sönmez, 2003, p.733). This mechanism is *direct*, i.e., students have to expose their preferences so that matchings are based on the revealed preferences and schools' priority orderings. As already stated, if schools are popular and oversubscribed, BSAM motivates students to misrepresent their preferences, mainly by indicating one of their lower preferences as their first choice (ibid, p.733f.). This will become clear when explaining the procedure. A school choice problem consists of two finite and disjoint sets, $students\ I = \{i_1, \dots, i_n\}$ and schools $S = \{s_1, \dots, s_m\}$, each with a capacity vector $q = \{q_{s_1}, \dots, q_{s_m}\}$. The students' preferences are denoted by a list $P = \{P_{i_1}, \dots, P_{i_n}\}$ and the priority ordering from the schools are denoted by $\pi = \{\pi_{s_1}, \dots, \pi_{s_m}\}$. Accordingly, a school choice problem is represented by the pair (P, π). Each student has to submit his preferences over all schools, and the priority ordering for each school s_k as the function $\pi_{s_k}: \{1, \dots, n\} \to \{i_1, \dots, i_n\}$ is generated according to four categories: If a student lives in the walking zone of school s_k, and has a sibling who is already enrolled at s_k, he has the highest priority $\pi_{s_k}(1)$. If only the latter is the case, students enjoy second priority $\pi_{s_k}(2)$ and if only the first is true, third priority $\pi_{s_k}(3)$. All other students have fourth priority $\pi_{s_k}(4)$ (c.f. Abdulkadiroğlu/Sönmez, 2003, p.733f. & Pathak/Sönmez, 2008, p.1638f.). There are multiple rounds in which students are matched to schools. "A matching $\mu: I \to S \cup I$ is a function such that

$\mu(i) \notin S \Rightarrow \mu(i) = i$ for any student i, and $|\mu^{-1}(s)| \leq q_s$ for any school s" (Pathak/Sönmez, 2008, p.1639). In the first round each school s_k assigns seats to the students with the highest priorities, but they are constrained by only considering those students who have listed s_k as their first choice. This is done until the school s_k has reached its capacity or until there are no students left who ranked s_k as their first preference. If s_k has not reached its capacity at the beginning of the second round, the school again considers the remaining students who ranked s_k as their second choice, and only them, and assigns seats according to the students' priority ordering. Again, this is done until s_k has reached its capacity or until there are no students left who ranked s_k as their second priority. The procedure continues accordingly until every student is matched to a school or until it is only the case that a student does not return an application or was not given any of his choices. Then he is assigned to a school that has not reached its capacity and is closest to the student's home (Abdulkadiroğlu et al., 2005b, p.369). However, according to Abdulkadiroğlu et al. (ibid), about 80 percent of students who participate in the first registration period are assigned to their stated first choice. Yet, this number can be misleading because the stated first choice is often not the most preferred school, as mentioned earlier. Nevertheless, Abdulkadiroğlu and Sönmez (2003, p.734) conclude that, if students stated their true preferences, the matchings resulting from BSAM would be Pareto-efficient.

The introduced concern, that unless a student lists a school s_k where he enjoys a high priority as his first preference, the student loses the priority to other students who have listed s_k as their first preference, is now specified with the help of the following example by Ergin and Sönmez (2006, p.216). Consider three schools a, b, and c with the quotas $q_a, q_b, q_c = 100$ and the set of students $N = \{I\} \cup \{J\} \cup \{K\}$, which is divided into the following subsets $I = \{i_1, \dots, i_{100}\}$, $J = \{j_1, \dots, j_{100}\}$, $K = \{k_1, \dots, k_{100}\}$.

The priorities of the schools are,	And the preferences of the students are
• $\pi_a = i_{1,\dots}, i_{100}$,	• $P(i_{1,\dots}, i_{50}) = a, b, c$. $P(i_{51,\dots}, i_{100}) = b, a, c$
• $\pi_b = j_{1,\dots}, j_{100}$,	• $P(j_{1,\dots}, j_{50}) = a, b, c$. $P(j_{51,\dots}, j_{100}) = b, a, c$
• $\pi_c = k_{1,\dots}, k_{100}$.	• $P(k_{1,\dots}, k_{50}) = a, b, c$. $P(k_{51,\dots}, k_{100}) = b, a, c$

If student i_{51} submits his true preference ordering b, a, c, his first choice school b assigns 50 of 100 seats to the students $j_{51,\dots}, j_{100}$ who chose school b first and have the highest priority there. The other 147 students competing for 50 seats at school b are $i_{52,\dots}, i_{100}$, $k_{51,\dots}, k_{100}$ and possibly $j_{1,\dots}, j_{50}$ who might misrepresent their preferences by indicating b as their first choice school as they know they have the highest chance for acceptance there. Therefore, it is very uncertain whether student i_{51} will be matched to school b. Yet, it is certain that his second ranked school a will reach its capacity at the first round because students $i_{1,\dots}, i_{50}$ are assigned to it (students' preferences equal school's priority) and $j_{1,\dots}, j_{50}$, $k_{1,\dots}, k_{50}$, as well as possibly strategic acting students $i_{52,\dots}, i_{100}$ compete for the remaining 50 seats. Thus, student i_{51} will not be considered at school *a* and risks being assigned his last preference school *c*.

For the example of school choice in Barcelona, Calsamiglia and Güell (2013) verify the assumption that under BSAM students might be strategic by stating their high-priority school as their first choice. There, schools in which students enjoy a high priority are often ranked as their first choice, and only few students are not assigned to them. Between the school years in 2006 and 2007 a change of priority zones was conducted, so that it is possible to identify whether preferring a school is correlated with it being a high priority school. Considering how often a school was stated as a first choice by families for which only the priority zone is important (no other factors as siblings are significant), a decrease of 68% was seen for schools which were a high priority school in 2006 but no longer in 2007. In the same vein, an increase of 400% was seen for schools having changed from a low to a high priority school in 2007.

Now, considering that it might be more efficient to misrepresent preferences, or actually that it is necessary to do so and that parents in fact do not have a *choice*, it is not surprising that local press advocates strategic behavior for BSAM. Also central agencies in charge of school choice programs e.g. the *Central Placement and Assessment Center in Minneapolis* and other stakeholder as for example the *West Zone Parents Group*[14] in Boston advise on strategizing (Chen/Sönmez, 2006, p.204f.). Hence most students do not state their true preferences; due to that BSAM becomes Pareto inefficient (Chen/Sönmez, 2006, p.208) and therefore, strategic students harm honest students (Pathak/Sönmez, 2008, p.1637). Having already mentioned that the University of Paderborn uses a mechanism based on BSAM, the matching task for which the mechanism is used is presented now. Given the introduced concern about a loss of efficiency due to strategic behavior, alternatives for BSAM will be introduced and examined afterwards.

University of Paderborn – Assigning students to supervisors

As stated on different section on the homepage of the University of Paderborn, the university has about 19,500 students in the winter term 2013/2014 of which about 4,000 students are enrolled in

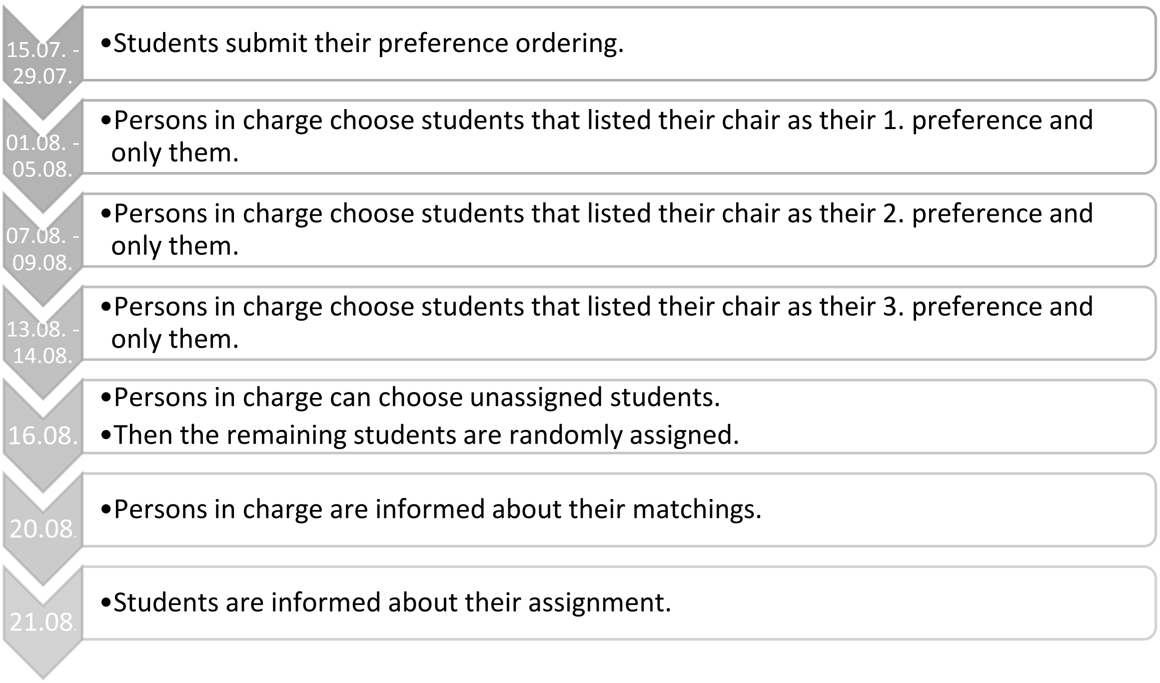
[14] Boston has three zones (East, West, North) in which there exist grades starting from kindergarten level and ending at the twelfth grade of high school (*K-12*) (Abdulkadiroğlu et al., 2005b, p.368). According to Pathak and Sönmez (2008, p.1636) the West Zone Parents Group had about 180 members in 2008, the group is still active, currently there are 1156 members (Yahoo!Groups: West Zone Parents Group, 2014), who concentrate on the K2 level and meet regularly to strategize on how to game the mechanism.

a program of the Faculty of Business Administration and Economics. Since the winter term 2012/2013 a web based central theses application procedure exists at the faculty. 36 Professors are currently working at six different departments of the faculty, four out of them are participating at the procedure. Namely, the departments *Management* (including Prof. Dr. Eggert, Prof. Dr. Fahr, Prof. Dr. Frick, Jun.-Prof. Dr. Iseke, Prof. Dr. Dr. h.c. Dr. h.c. Rosenthal, Prof. Dr. Schnedler, Prof. Dr. Martin Schneider, Prof. Dr. Wünderlich and since the winter term 2013/2014 also Prof. Dr. Kabst,), *Taxation, Accounting, Finance* (including Prof. Dr. Betz, Prof. Dr. Schaper, Prof. Dr. Schiller, Prof. Dr. Dr. Georg Schneider, Prof. Dr. Sureth, Prof. Dr. Werner, and since the winter term 2013/2014 also Prof. Dr. Uhde), *Economics* (including Prof. Dr. Feng, Prof. Dr. Gilroy, Prof. Dr. Gries, Prof. Dr. Haake, Prof. Dr. Hehenkamp, Prof. Dr. Kraft, Prof. Dr. Jungblut) and *Business and Human Resource Education* (including Prof. Dr. Beutner, Prof. Dr. Kremer, Prof. Dr. Sloane, Prof. Dr. Winther, Prof. Dr. Gerholz) as well as Prof. Dr. Niclas Schaper (Chair for industrial and organizational psychology) (Universität Paderborn, 2014a,b,c,d,e,f).

The mechanism is aimed at fairly distributing the workload of supervising theses according to the capacity of each chair for every semester. Students' preferences should be respected at the same time. The matching process takes no longer than two months and the student will be informed via email afterwards. The assigned person in charge from every chair can constantly see all students' preferences after they are uploaded to SharePoint.[15] In addition to that, the persons in charge must be available throughout the whole process. This is a major aspect for the feasibility of the mechanism, thus, it will be discussed again at a later point. The only advice on the procedure for students is to regularly check emails and to choose chairs specialized in topics in which the student is interested in. This should ideally be reflected in previous course choices. Yet, there is no advice on how to strategically avoid the deficiencies of the mechanism by misrepresenting one's preferences. In *Figure 6* it is illustrated how the process was structured for writing a thesis in the winter term 2013/14, the resemblance to BSAM should be noted (c.f. Hoyer, 2014; Universität Paderborn, 2014b,g).

[15] Share Point is a product from Microsoft that, according to their information, provides the user "a secure place to store, organize, share, and access information" (Microsoft Office, 2014)

Figure 6: Mechanism at UPB for assigning students to supervisors



Source: Own representation based on Hoyer, 2014

As explained at the beginning, a perfect matching should be achieved. Therefore, the number of how many students a chair needs to supervise depends on the overall number of theses and researchers. This will now be explained more detailed. After all preferences are submitted it is calculated how many Bachelor and Master Theses need to be supervised. A Bachelor thesis is worth one point and a Master thesis 1.5 points, the sum of points per semester is calculated accordingly. Furthermore, the number of full time equivalent research positions each chair contains,[16] is known. Given that the sum of points for a semester is p and the number of researchers employed at chair h in that semester is r_h and the total number of employees is r, the quota[17] for chair h is defined as $q_h = \left(\frac{p}{r}\right) * r_h$ (c.f. Hoyer, 2014).

Anonymized data on students' preferences, their assignment and how many times they participated in the procedure is available for the summer term 2013 and the winter term 2013/14. In addition to that also the number of researchers at each chair, to which degree the chairs fulfilled their quota and the quota itself is known. For the winter term 2012/13 only the information on the chairs (quota, number of assigned students, and number of researchers) is available, thus, all following conclusions derived from data only refer to the 611 matchings of 590 students in the summer term 2013 and the winter term 2013/14.[18]

16 Those researchers that are hired through external funding without teaching obligations are not taken into account. From here on, the term researchers only describes full time equivalent research positions.

17 Whenever $\left(\frac{p}{r}\right)$ does not result in an integer, it is rounded up.

18 This is unambiguous data, special cases or uncertain information are excluded.

As explained by Roth (1985) a distinction between one-to-one matching and one-to-many matching is that in the college admission problem an outcome exists that all colleges strictly prefer to O^c and no stable matching procedure makes it a dominant strategy for every c to state its true preferences. In context of school choice these aspects are neglected as it is assumed that colleges are no strategic actors. Yet, as mentioned, the matching mechanism at UPB is established to also benefit the chairs. Moreover, supervisors do have preferences on which students they want to work with and which students they evaluate as suitable for their chairs. One can assume that these preferences are responsive and substitutable, thus they meet the earlier introduced prerequisites by Roth (1985). In order to analyze if these preferences correspond to the college admission problem or to school choice, a survey was conducted. It was performed online on the website *Survey Monkey* to guarantee the anonymity of all respondents. Out of the 28 participating chairs 24 answered the survey on how they select students, this equals a response rate of almost 86 percent.[19] The persons in charge arranged an order of six given factors. The order reflects the importance of these factors for students, in order to obtain a high priority at the chair: Chosen courses (concentration on one area of expertise, having completed courses at the chair), average grade (of all completed courses), grades in related courses (only courses of one area of expertise), resume (other qualifications as internships, work experience, etc.), documented interest and motivation (conversation, letters of motivation, direct contact), and other factors (that could be stated). As a conclusion from the received answers one can assume a trend towards the following priority ordering, the proportion of chairs who ranked the factor at that position is stated in brackets:

1. Chosen courses (50%)
2. Grades in related courses (50%)
3. *Documented interest and motivation (25%)*
4. Average grade (41.67%)
5. *Resume (45.83%)*
6. Other factors (75%)

It should be noted, that especially for the *documented interest and motivation* the opinion varies (25% rate it as the third priority, 25% as the second priority, 20.83% as the fifth priority), the same occurred for the factor *resume* (41.67% rated it as the fourth priority, 25% rated it as the fifth priority). The general trend towards categorizing aspects as more important (one to three) or less important (four to six) is about the same for most chairs.[20] Consequently, the context of UPB can

[19] The actual response rate can be considered even higher as one person is in charge of three chairs. Because the survey was conducted anonymously, it is not possible to distinguish the answer of this particular person, so this aspect was not taken into account.
[20] The survey and all specific results can be found in the appendix.

be considered equivalent to school choice, in which college's priorities are also determined exogenously.

To continue by focusing on the performance of the mechanism at UPB, Chen and Sönmez (2006, p.209f.) note that BSAM is frequently advertised by the fact that it often accommodates students' first preferences. However, a result of an experiment conducted by Chen and Sönmez (2006), which will be presented now, shows that instead of 70.8 percent of the participants, the low percentage of 28.5 received their *true* top choice, the significantly higher percentage was only assigned their *stated* first choice (ibid, p.216).

Performance of the mechanism at UPB

Based on the just introduced data, the mechanism at UPB provides the following statistics: 75.61% of all matchings resulted in the student's first choice, 11.62% in the second choice, 5.73% in the least preferred chair and 7.04% were randomly assigned or chosen by a chair after the third round. Chen and Sönmez (2006, p.204f.) oppose that empirical data which is derived from stated preferences cannot assess the efficiency of a mechanism. The objection is based on an experiment which they conducted to analyze the efficiency of BSAM and two alternative mechanisms, which will be introduced later. Important in this controlled laboratory experiment[21] is that the *designed environment* is constructed in a way that students' preferences are correlated with the quality of schools and their proximity.[22] The payoff for each student depends on whether the students' preferences are met in the matching. In the *random environment* students' preferences and therefore their payoff for each school is chosen randomly (ibid, p.211f.). As a conclusion from this experiment, Chen and Sönmez (2006) suggest that the two alternatives can improve efficiency when compared to BSAM.

One aspect analyzed by Chen and Sönmez is the question when students misstate their preferences. As one result they found that under BSAM the capacity of their true first choice affects the proportion of truthful preference revelation. Namely the smaller the capacity was, the more preferences were misrepresented. In contrast, when a student's first choice was a small school and the alternative mechanisms were applied nothing alike was detected. In the same vein, Chen and Sönmez (2006, p.219ff.) additionally identify that under BSAM about two-thirds of the students state a school as their first choice that is more realizable instead of their actually preferred school,

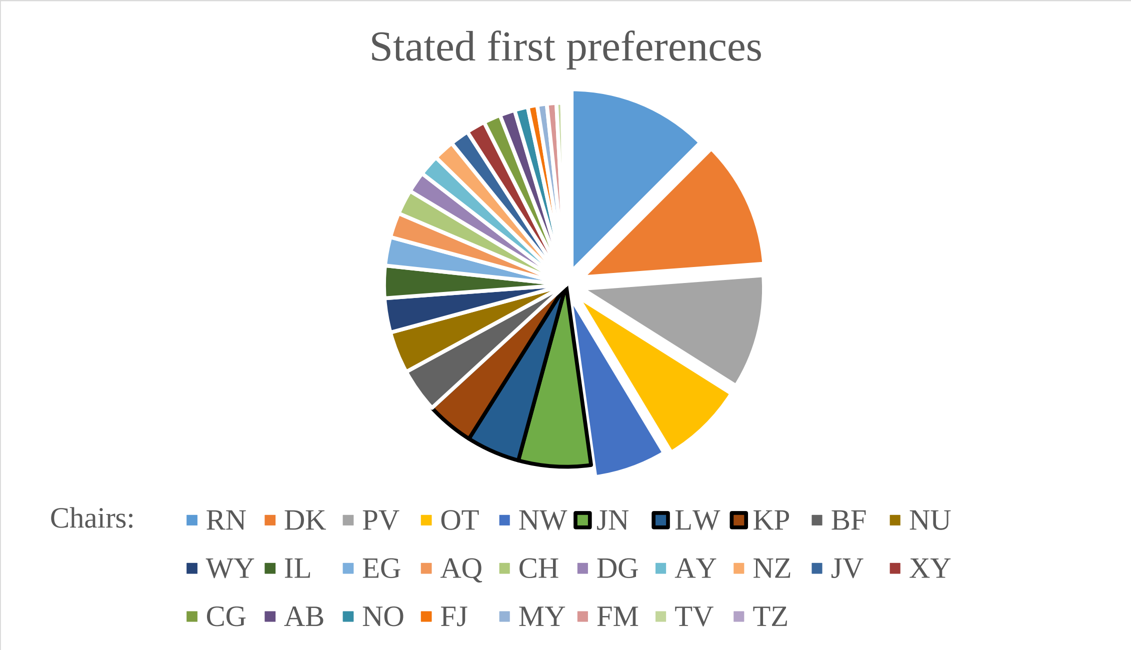
[21] A 3 x 2 design, each mechanism is examined in a designed and a random environment. "For all treatments in each session, there are 36 students and 36 school slots across seven schools. The schools differ in size, geographic location, specialty and quality of instruction in each specialty" (Chen/Sönmez, 2006, p.211).

[22] There are students that prefer the specialty science and others who prefer arts. Students' preferences are determined by a utility function which depends on the proximity of the school, the quality of school regarding the specialty (arts or science) and a random factor (ibid).

confirming the results from Barcelona that were already presented (Calsamiglia/Güell, 2013). In addition to that, almost 19 percent use the *Minneapolis strategy* by "making the first choice a true favorite and the other two 'realistic'" (Chen/Sönmez, 2006, p.219). The average payoff of this strategy in the designed environment does not deviate from other strategies but it achieves considerably higher payoffs in the random environment. Chen and Sönmez ascribe this to different levels of competition. The situation is less competitive in the random environment because less students have a popular first choice so there is little conflict of interest between the students. In that case, generally only the first preference is relevant as it can often be satisfied. Then, BSAM must not be inefficient. However, in the designed environment the complete preference ordering is relevant. As there is much conflict of interest in the current situation in Boston, in reality significant efficiency losses can be assumed if the Minneapolis misrepresentation is used (ibid, p.219ff.).

Having identified that the level of competitiveness which can be related to the capacity of schools is an important factor, it is necessary to assess the situation in Paderborn accordingly. In the context of the University of Paderborn one can assume that the capacities are not known by the students. As explained earlier, only after all applications for theses are submitted it is calculated which chair has to supervise how many students. Nevertheless students know which chairs are especially popular and how many people work there. Although, as explained earlier, the latter does not correspond perfectly to the number of researchers, students approximately know which chairs tend to be strongly demanded. Hence, this aspect can be a major factor for a possibly high rate of misrepresentation at UPB. Unfortunately, there is no data on the real preferences of UPB students, neither exists data on the major of the students so one cannot conclude which chairs must be preferred more often over chairs with unpopular areas of expertise. So the question whether UPB represents a more competitive or a less competitive environment should be analyzed more detailed. It is obvious that there always exist chairs that are outstandingly popular. Looking at the data, the relation between how many first choice students a chair could supervise and the quota of how many students it needs to supervise proves this suggestion. This is now presented in detail. Looking only at the stated *first* preferences of the students *Figure 7* illustrates that there is an unambiguous tendency towards stating one out of five chairs as a first choice. The five most popular chairs (enlarged parts in *Figure 7*) are ranked as a student's first choice in 47.8 percent of all considered cases (283 out of 592 stated first preferences). Looking at the two most popular chairs, they are ranked as a student's first choice in 23.82 percent. The most popular chair, indicated in light blue in *Figure 7* is ranked as a student's first choice in 12.5 percent of all cases. This indicates a very high level of competition. Yet, what has to be taken into consideration is that the quota for each chair is different. The fact that the same chairs are ranked the highest by most students (enlarged in *Figure 7 and 8*) can be compensated if these chairs have to fulfill a higher quota. According to the data, the five most popular chairs belong to the ten (summer term 2013) or twelve (winter term 2013/14) biggest (, i.e., greatest quota) chairs. For example the most popular chair is the second biggest one in the summer term 2013, as it has 4 researcherss, exactly like the third most popular chair and another one. In *Figures 7*, *8* and *12* the chairs are indicated by the same colour.

Figure 7: Popularity of the chairs at UPB

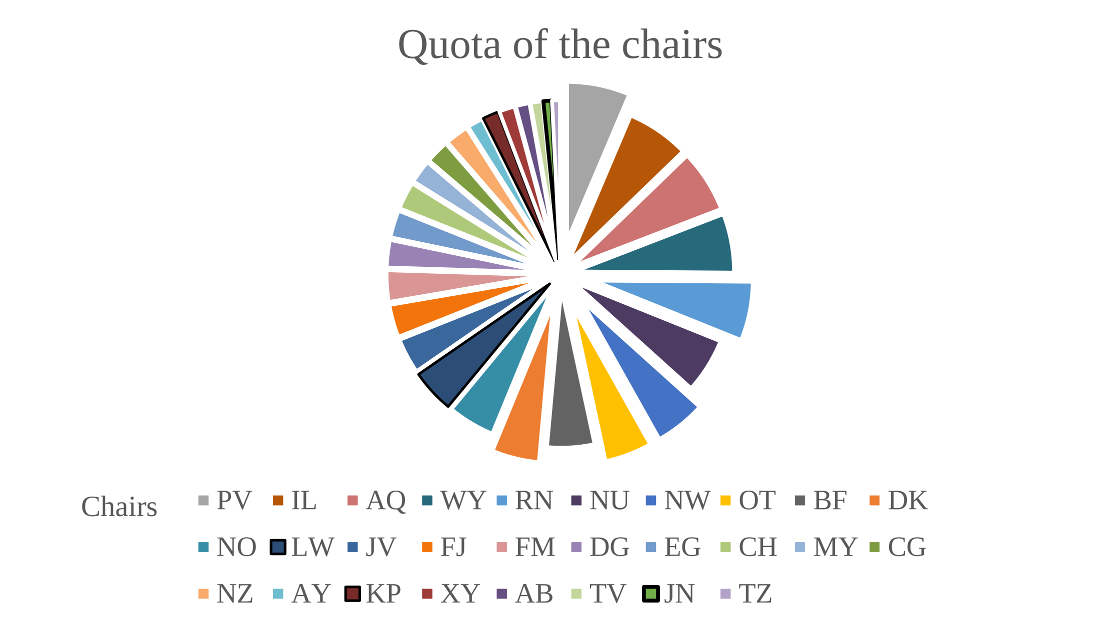


Source: Own representation

Figure 8 indicates that the high demand for some chairs can be compensated to a small degree by a higher quota, but it also shows that other chairs that are averagely popular have a very small quota, examples are framed in black in *Figure 7 and 8*. To specify this problem, the relation between how many first choice students a chair could supervise and the quota of how many students it needs to supervise is analyzed.

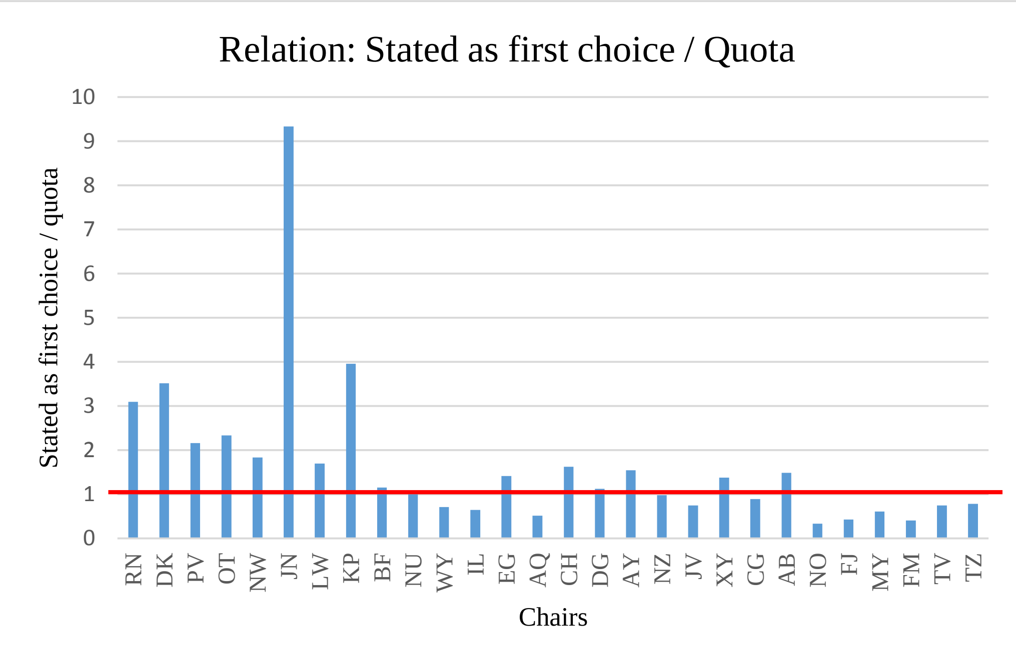
The relation indicates that the situation in Paderborn is rather competitive, this is illustrated in *Figure 9* in which a value of one would be optimal, and the higher the bars the higher is the competitive level.

Figure 8: Quota of the chairs at UPB



Source: Own representation

Figure 9: Relation of the chairs being stated as first preference and the quota they have to fulfill



Source: Own representation

Based on the experiment by Chen and Sönmez (2006) it should be assumed that this competitive environment results in the frequent use of strategies like the Minneapolis strategy or high misrepresentation of the first choice. Therefore, also at UPB the mechanism provides a high incentive to not submit preference orderings corresponding to true preferences in order to avoid the risk of being assigned randomly. Thus, the data was analyzed with respect to other aspects indicating the same. One category that was analyzed in detail is the group of students who were not assigned their first or second choice. Pathak and Sönmez (2008, p.1636) state that under BSAM in four years in Boston roughly 20 percent of students stated two highly demanded schools as their first two choices, and 27 percent of them ended up unmatched and were assigned to a school close to their home that had not reached its capacity yet. The data from UPB shows that there are 43 matchings (out of 611) in which the result was a random assignment of the student, this equals about seven percent of *all matchings*. From these seven percent about *79 percent* (34 out of 43) had ranked one of the five popular chairs as the first choice chair. In addition to that 5.73 percent of *all matchings* resulted in an assignment to the student's last preference. 16 out of these 35 matchings, i.e., *45.71* percent had also one of the five popular chairs as the first preference. This indicates that the students who did not perform well under the mechanism, mostly had a competitive first preference and stated their true preferences.

Referring to the context of school choice in Barcelona again, the majority of students there does not act very risky i.e. stating a high priority school as their first choice, but 20% of all students chose the outside option of going to a private school. These students, enjoying the possibility of having another option when being assigned randomly, can act riskier (Calsamiglia/Güell, 2013, p.12). Applying this factor to the context of UPB, it is interesting to analyze another category of students, namely when they have an outside option. In the context of UPB, this is the case when students participated twice in a row. [23] 20 students out of 591 participated twice and for 20% out of the 20 students (so for four students) it is both valid that they were randomly assigned a chair in their first participation and that they had one of the popular chairs ranked as their first choice in their first participation. Additionally, two more students who participated twice had chosen one of the popular chairs as their first choice and were not assigned to it but to one of their second or third choices in their first participation. A suggestion is that these students might not have been satisfied enough thus they decided to wait a semester and try again. Their dissatisfaction with the procedure can be related to them having a competitive first preference and still stating their true preferences. As a conclusion from these results, one can assume that students at UPB are not satisfied with the mechanism, thus, that the mechanism is not ideal. Chen and Sönmez (2006, p.204) summarize that BSAM is neither efficient nor stable and in addition to that it generates incentives to misrepresent preferences. Furthermore we have just shown that UPB is a competitive environment in which honest students suffer under the mechanism that is based on BSAM. Thus, two mechanisms will be presented now, to see whether they could be suitable alternatives for the mechanism used at UPB.

[23] For future research it would be worth expanding the data set regarding how many ECTS each student has when he participates in the procedure as this can roughly signal whether the student will be enrolled at UPB for more than one semester, and could, thus, postpone writing the thesis when he is not assigned to his preferred chair.

Alternatives for the Boston Student Assignment Mechanism

Abdulkadiroğlu and Sönmez claim that in their paper *School Choice: A Mechanism Design Approach* (2003, p.732f) they are the first to approach the school choice problem by proposing two alternative mechanisms to BSAM. Firstly the deferred acceptance mechanism by Gale and Shapley that was analyzed in detail above, and secondly the *Top Trading Cycles Mechanism* (TTC). Both alternatives have not been applied in reality until recently.

To explain TTC shortly before introducing the mechanism later in detail, it can be said that it creates a "virtual exchange for priorities" (Abdulkadiroğlu et al., 2005b, p.370). Consider a group of students who each have the highest priority at one school but prefer another. By simply allowing them to trade their priorities, efficiency is improved for the students (ibid). This mechanism as well as the deferred acceptance mechanism "have superior theoretical properties" (Chen/Sönmez, 2006, p.202) when compared to BSAM. Hence, Chen and Sönmez discuss these matching mechanisms and emphasize that they exemplify two different approaches on the "trade-off between elimination of justified envy and Pareto efficiency" (ibid, p.209). Before examining the applicability of TTC and the deferred acceptance mechanism to the situation in Paderborn, the two mechanisms will be presented (again).

Deferred Acceptance Mechanism

The mechanism has been explained in detail in context of the marriage market, owing its name to the fact that assignments are done tentatively before the matching is ultimately accepted (Abdulkadiroğlu et al., 2005b, p.370). The mechanism is successfully applied in New York City for high schools (Abdulkadiroğlu et al., 2005a) and the Boston School Committee changed their mechanism to the deferred acceptance mechanism in the school year of 2005/06 (Pathak/Sönmez, 2008, p.1636). To briefly summarize it, the most important attributes of the mechanism are that it is strategy-proof and results in an outcome which can only be Pareto-dominated by an *unstable* matching. An important advantage of this mechanism is that its outcome is stable, for reality it is important that justified envy is eliminated. This is because instability can lead to lawsuits from dissatisfied students, as it results in a situation in which a child with lower priority is admitted and another child with higher priority is excluded (c.f. Abdulkadiroğlu et al., 2005b, p.371 & Chen/Sönmez, 2006, p.204). Complete elimination of justified envy is for example imposed by law regarding university admission in Turkey (Balinski/Sönmez, 1999).

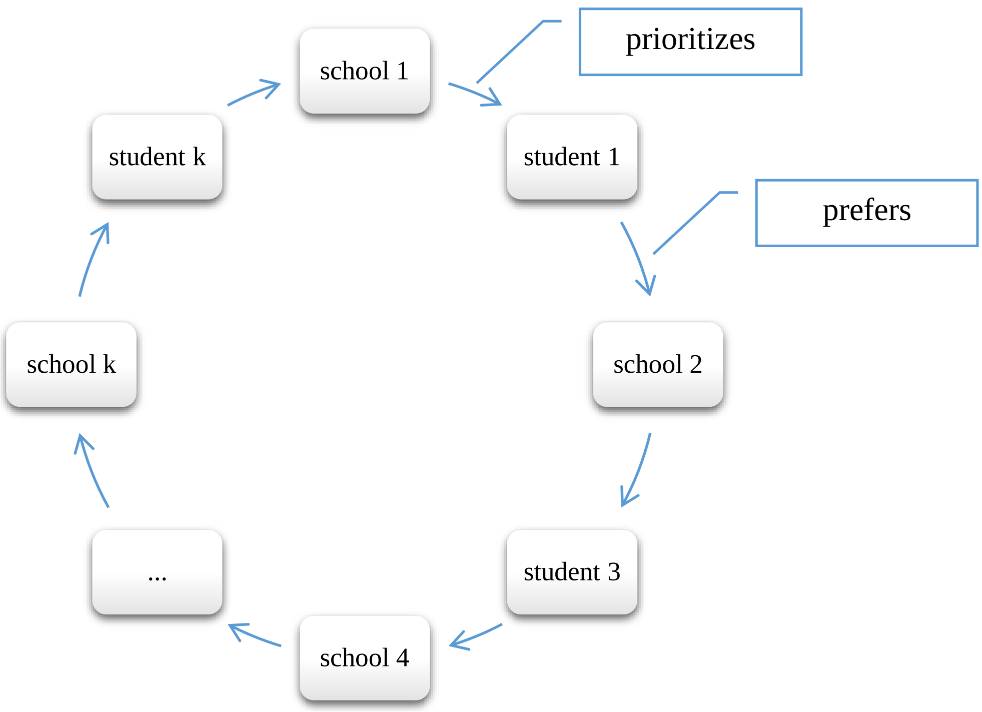
Top Trading Cycle Mechanism

The Top Trading Cycle Mechanism (TTC) is based on an algorithm attributed to David Gal, initially developed in the context of housing markets (c.f. Shapley/Scarf, 1974), and is also a direct mechanism. TTC is Pareto efficient, but as shown, this is incompatible with stability, thus, there is no complete elimination of justified envy (Chen/Sönmez, 2006, p.208). In TTC, as mentioned above, the priorities of students are seen as an opportunity to be accepted by a school (Abdulkadiroğlu/Sönmez, 2003, p.736). Given the example by Chen and Sönmez (2006, p.209f.), there are two finite and disjoint sets of students and schools, for which the priority ordering is defined in a way that students, living in the walking zone of the school, are guaranteed high priority and all other students are assigned low priority. Hence, a school in which a student enjoys high priority is his *district school.* The priority ordering among the two categories (high and low priority) is determined by a lottery. Each student has strict preferences over each school and submits his preference ranking. Then everyone is tentatively assigned to his district school. Next the lottery determines an order for all students who line up accordingly in a queue. The first one in the queue is asked for his top choice school, if it is his district school the tentative assignment becomes certain and the student leaves the queue, i.e., the assignment process. If he ranks a school *S* higher than the tentatively assigned district school, the first student in line who is tentatively assigned to this school *S* (i.e. whose district school is school *S*) moves up the queue so that he is first in line. At that point the process is repeated again until there is a cycle. According to Abdulkadiroğlu and Sönmez (2003, p.737) a cycle is a group of students $(i_1, i_2, \dots, i_k)$ and schools $(s_1, s_2, \dots, s_k)$ in which s_1 prioritizes i_1 but i_1 prefers s_2, then again s_2 prioritizes i_3 but i_3 prefers s_4,…, s_k prioritizes i_k but i_k prefers s_1. As illustrated in *Figure 10* this is a situation in which an exchange is Pareto-improving, so each student is assigned to their preferred school. If the capacity of a school is reached after a circle, the school will exit the process. The process is finished when all students are assigned a school (Chen/Sönmez, 2006, p.210).

Exactly like the deferred acceptance mechanism, TTC is also strategy proof. As just explained a student always chooses his most preferred school. So given that the student leaves the algorithm at step *t*, the school he is assigned to at step *t* is either his first preference or all more preferred schools have left the algorithm before step *t.* This would also have happened if the student had misrepresented his preferences (Abdulkadiroğlu/Sönmez, 2003, p.738).

There exist variants of TTC, for example a model of house allocation by Abdulkadiroğlu and Sönmez (2003, p.737) that inserts already existing tenants at the top of priority orderings. In other contexts, certain quotas need to be taken into consideration, like racial quotas. These aspects are neglected in this paper as the mechanism applied in Paderborn does not need to take any comparable aspects into account. Similarly, wait-lists are frequently used in some matching mechanisms, this feature is neglected as well, out of the same reason. Here it should be noted that at UPB a chair fulfills its quota by being matched to students, not by actually supervising them, as students are free to resign from the matching until formally having registered for the thesis at the examination office. Hence, despite resignations a chair fulfills its quota whereby waiting lists become unnecessary.

Figure 10: Top Trading Cycle



Source: Own representation

Applicability of the mechanisms to the context of UPB

Table 13 summarizes the earlier mentioned superior theoretical properties of the two alternative mechanisms in contrast to the deficiencies of BSAM.

Table 13: Comparison of the three mechanisms

	BSAM	Deferred Acceptance Mechanism	TTC
Stability	No	Yes	No
Pareto Efficiency *(only referred to the welfare of students)*	No	No[24]	Yes
Truthfully stated preferences	No	Yes	Yes

Source: Own representation

[24] As explained before, it is only “constrained-efficient among mechanism that eliminate justified envy” (Chen/Sönmez, 2006, p.202).

The experiment of Chen and Sönmez (2006) is consulted again to look at the results of the alternative mechanisms. Chen and Sönmez' first result is in accordance with the third row of *Table 13*. Over 80% of the contestants misrepresent their preferences under BSAM (ibid, p.227). In addition to that they can specify *(Table 14)* that in the designed environment the proportion of truthful preference revelation under TTC is significantly lower (56.3 percent were assigned their reported top choice, 31.3 percent their true top choice) than the proportion under the deferred acceptance mechanism (51.4 percent were assigned their reported top choice, 35.4 percent their true top choice). In the random environment the proportion of truthful statements is weakly lower (ibid, p.215ff.).

Table 14: Comparison of truthful preference revelation under the three mechanisms

	BSAM	**Deferred Acceptance Mechanism**	**TTC**
Proportion of truthful preference revelation	Low	High	Medium

Source: Own representation

According to Chen and Sönmez (2006, p.227f.), the different competitiveness levels in the environments are one major source for surprising results on the efficiency of the three mechanisms which are summarized in *Table 15*. The result that BSAM can be as efficient in a less competitive environment as the deferred acceptance mechanism has been discussed. Having shown that the situation at UPB is rather competitive, the designed environment is of greater importance. *Table 15* shows that TTC did not result in the expected high efficiency level, in both environments.

Table 15: Comparison of Pareto efficiency of the three mechanisms

	Designed Environment (competitive environment)	**Random Environment** (less competitive environment)
Pareto Efficiency	Deferred Acceptance > TTC > BSAM	BSAM ~Deferred Acceptance > TTC

Source: Own representation

An explanation for this result, according to Chen and Sönmez (ibid, p.219), is the high number of misrepresentations under TTC. Not only under TTC, also under the deferred acceptance mechanism, misapprehension over the mechanism at the side of the students is very likely to result in an efficiency loss. Therefore, the US Office of the Educational Research and Improvement (1992, p.19) emphasizes the factor *Instructional Capacity* as an important factor for efficient results. Many students do not understand the dominant strategies for the deferred acceptance mechanism and TTC, especially when the instructions are more complex as it is the case with TTC. In order to reduce efficiency loss, Chen and Sönmez recommend to educate students on "the strategy proofness" (2006, p.228f.) of the two mechanisms. As a conclusion the effort (especially time and educational expense due to the high number of participants) that is necessary to apply TTC successfully at UPB is extremely high, thus, TTC is not considered as a suitable alternative for UPB. The evidence from the experiment on the efficiency of the deferred acceptance mechanism and BSAM is mixed so the two mechanisms will now be thoroughly examined in theory again.

Pathak and Sönmez compare the mechanisms in their paper *Leveling the Playing Field: Sincere and Strategic Players in the Boston Mechanism* (2008). As has been stated already, strategic students harm sincere students under BSAM, thus, a "*preference revelation game*" (Pathak/Sönmez, 2008, p.1639) emerges which was analyzed by Pathak and Sönmez under the name *Boston game*. Given the above defined context of school choice with a set of three schools $S = \{a, b, c\}$, the capacity vector $q = \{1,1,1\}$ and the priority orderings and students' utilities:

- $\pi_a = i_2, i_1, i_3$
- $\pi_b = i_3, i_2, i_1$
- $\pi_c = i_2, i_3, i_1$

Table 16: Students' utilities

	a	b	c
u_{i_1}	1	2	0
u_{i_2}	0	2	1
u_{i_3}	2	1	0

Source:
Own representation based on Pathak/Sönmez, 2008, p.1640

Furthermore, there is the set of three students $I = \{i_1, i_2, i_3\}$ of which the first two are willing to act strategically and misrepresent their preferences. Their preferences $P = \{P_{i_1}, P_{i_2}, P_{i_3}\}$ are denoted by the utilities in *Table 16*. Student i_3 will always state $\{abc\}$ but the strategy space for $i_1$and i_2 is $\{abc, acb, bac, bca, cab, cba\}$, the according $6 \times 6 \times 1$ Boston game ($i_1$being the row player) is:

Table 17: Boston game

	abc	acb	bac	bca	cab	cba
abc	(0,0,1)	(0,0,1)	**(1,2,0)**	**(1,2,0)**	(1,1,1)	(1,1,1)
acb	(0,0,1)	(0,0,1)	**(1,2,0)**	**(1,2,0)**	(1,1,1)	(1,1,1)
bac	(2,0,0)	(2,0,0)	(0,2,2)	(0,2,2)	(2,1,2)	(2,1,2)
bca	(2,0,0)	(2,0,0)	(0,2,2)	(0,2,2)	(2,1,2)	(2,1,2)
cab	(0,0,1)	(0,0,1)	(0,2,2)	(0,2,2)	(0,2,2)	(0,2,2)
cba	(0,0,1)	(0,0,1)	(0,2,2)	(0,2,2)	(0,2,2)	(0,2,2)

Source: Own representation based on Pathak/Sönmez, 2008, p. 1641

The Nash equilibrium profiles are highlighted in bold in *Table 17* and the result of this "coordination game among strategic students" (Pathak/Sönmez, 2008, p.1637) is a Nash equilibrium which each student weakly prefers to any other equilibria assignment. Pathak and Sönmez refer to this as the "*Pareto-dominant Nash equilibrium outcome*" (ibid, p.1643) which is highlighted in bold in *Table 18* below. Student i_3 (P(i_3) $= a, b, c$) is assigned school c in the Pareto-dominant Nash equilibrium outcome although i_3 prefers school b and has the highest priority there. As represented in italics in *Table 18*, the honest student i_3 would be better off under the deferred acceptance mechanism. Student i_1 is indifferent between the results from the two mechanisms, whereas student i_2 prefers BSAM over the deferred acceptance mechanism. The deferred acceptance mechanism results are indicated in italic, blue-highlighted characters in *Table 18*. Hence, looking at the Pareto-dominant Nash equilibrium outcome a student is weakly better off when he is strategic, yet all other strategic students would weakly prefer the student to be sincere as they then have a competitive advantage over him (Pathak/Sönmez, 2008, p.1638 & p.1646).

Table 18: Comparison: Pareto-dominant Nash equilibria and deferred acceptance mechanism

$P(i_1)$ (strategic)	b	***a***	c
$P(i_2)$ (strategic)	**b**	*c*	a
$P(i_3)$ (honest)	a	*b*	**c**

Source: Own representation

Comparing BSAM and the deferred acceptance mechanism in school choice, Pathak and Sönmez (2008, p.1641ff.) prove that under BSAM, the set of Nash equilibrium outcomes, called *Nash equilibria of the Boston game* (ibid, p.1637) is equivalent to the set of matchings resulting from the deferred acceptance mechanism of a school choice problem (P, π) with a modified priority structure π^*, in which sincere students have the lowest priority for all but their most preferred school. Indeed it was shown earlier that under BSAM, sincere students, when compared to strategic ones, will receive a lower priority at schools they have not ranked first (ibid, p.1642). So to not favor strategic students the deferred acceptance mechanism can be used to replace BSAM because as presented above strategic students weakly prefer their assignments under the Pareto-dominant Nash equilibrium outcome from BSAM[25] to their assignments resulting from the deferred acceptance mechanism (ibid, p.1646).

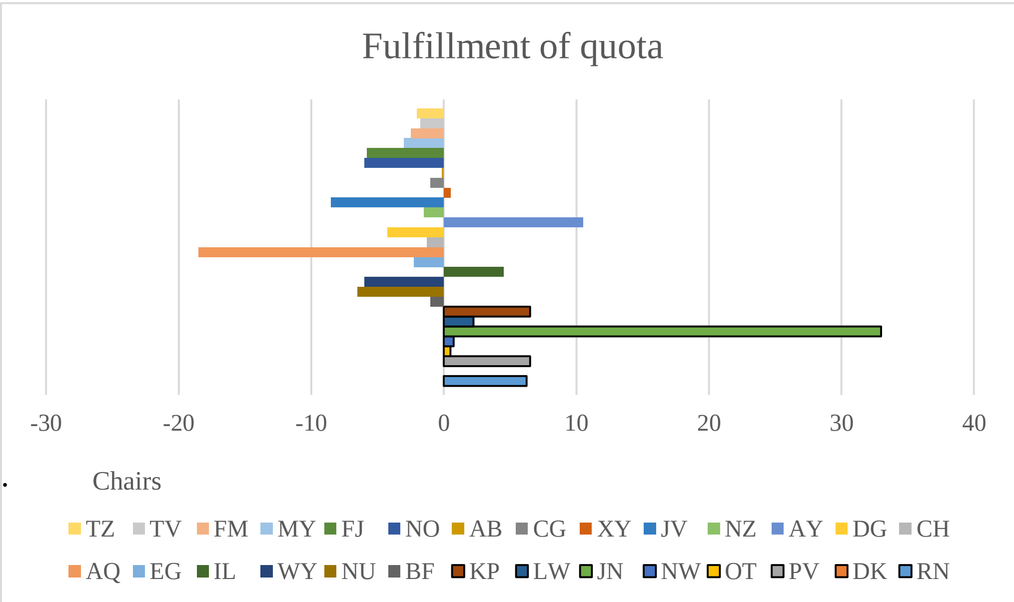
All in all, applying this to the context of UPB, it is better to apply the deferred acceptance mechanism instead of BSAM when all students are strategic, as then none has a competitive advantage any more (ibid, p.1638). It has been proven theoretically, that the situation that all students are honest is highly unlikely. This was also verified by statistics from UPB.[26] In the situation in which there are strategic as well as honest students, it is desirable to punish strategic

[25] Note that this is only the case for the Pareto-dominant Nash equilibrium outcome, it does not extend to all Nash equilibria. Yet, “computational experiments suggest that multiplicity is not a significant problem” (Pathak/Sönmez, 2008, p.1645)

[26] Theoretically, there is the possibility, that sincere students, when compared to other sincere students, can prefer their outcome under BSAM to the outcome of the deferred acceptance mechanism. Pathak and Sönmez (2008, p.1644) show an example in which a sincere student i_1 gains advantage under BSAM for his k^{th} school choice if sincere student i_2 ranks the school as his k+1 or lower choice. Yet, student i_2 then prefers the deferred acceptance mechanism

students with less preferred outcomes, hence, applying the deferred acceptance mechanism as explained above. Thus, theoretically the University of Paderborn should use the deferred acceptance mechanism because as a public institution UPB should encourage ethically correct behavior. Practically this would be inefficient. Imagine that the context of UPB would be a one-to-one matching of 36 students to 36 professors (which represents not even 10% of the students that participated in each semester), according to Gale and Shapley (1962, p.12) the maximum amount of stages that would exist for this example is 1262 ($36^2 - 36 + 2$). The much easier mechanism currently in use extends over 1.5 months even now, which is already problematic (c.f. Hoyer, 2014). A change towards a more time consuming mechanism would not be appreciated, although the new mechanism would result in preferred outcomes. Thus for reasons of feasibility, i.e. reasons of efficiency, it is recommended to improve the current mechanism at UPB instead of replacing it. There already exist incentives for chairs to supervise more students than their quota requires them to, as for example a compensation for the additional workload ("Belastungsausgleich"). It guarantees the chair additional research funds, if the chair exceeds its quota[27]. Such factors should be supported and complemented. As can be seen in *Figure 11*, many students already benefit by this, as most of the five most popular chairs and the previously discussed chairs, that are averagely popular but have a small quota, (framed black) exceed their quota (positive length of bar). The more popular the chair, the closer it is positioned to the abscissa, a chair that meets its quota perfectly is not represented by a bar, as this is represented by the value zero.

Figure 11: Fulfillment of quota



Source: Own representation

[27] The exact amount is endogenously determined, depending on the number of theses of the semester. Again a distinction is made between Bachelor- and Master Theses, as the latter is rewarded according to the additional efforts needed.

Conclusion

The theory of matching applies differently to various contexts, the more detailed and multifaceted assignments become, the more carefully simple definitions, assumptions and implications have to be reviewed and adapted. When looking for an optimal assignment, one of the most important aspects is to decide if stability is of great importance. Thus, a matching mechanisms that results in stable assignments was presented. For contexts in which justified envy is not a concern, an alternative mechanism that results in Pareto dominating assignments was introduced as well. Despite the existence of the deferred acceptance and the top trading cycle mechanism, BSAM is mostly used. Its deficiencies were illuminated to emphasize the assumption that the similar mechanism applied at UPB might not be ideal. Empirical evidence has shown that matching mechanisms, even when possessing superior theoretical properties, can be less efficient when applied to real matching assignments. As a conclusion, when selecting a mechanism, the central issue is not the trade-off between stability and Pareto efficiency, but first of all a trade-off between efficiency costs (i.e. feasible procedures) and the characteristics of matching mechanisms in general (stability, Pareto efficiency, incentives to misrepresent preferences). Yet, one should be able to prioritize these positive characteristics *without* suffering efficiency losses.

This thesis has shown that the assignment of students to supervisors at UPB is done in a competitive environment in which students are very likely to misrepresent their preferences. To change this, the two alternative mechanisms could be implemented theoretically, but due to the immense educational efforts, the time required, and the organizational complexity, changing to these alternatives will not be an efficient solution. Thus, it is advisable for the future to invest into examining possible features for BSAM that can also be implemented at the current mechanism at UPB. Another aspect that is worth investing in, is enriching the information on the agents who are matched. The preferences of the supervisors should be monitored to assess the finding that they are determined exogenously. Furthermore, to advance the analysis of the satisfaction with the mechanism, participating students should provide more information on their study progress. As the number of semesters can be misleading, each student should provide his number of ECTS. If only few ECTS are missing to complete the degree, it can be assumed that the student is not planning on staying at UPB for more than one semester. If more ECTS are missing, it can indicate that the student enjoys the outside option of participating for a second time. Unfortunately, students who misrepresent their preferences by stating a “safer choice” above their real choice would never reveal their true preferences as this could upset the chair that was strategically ranked first instead. Unfortunately an enrichment of data can only minimize this difficulty, this aspect will remain an issue for further proceeding in this matter.

As a conclusion it can be said, that a choice should remain a choice. Hence, if it is ignored that under a more feasible mechanism, participants are forced to strategically downgrade their preferences, the possibility of actually *choosing* something is eliminated. Matching mechanisms are supposed to match real preferences in the most efficient way, research should aim at realizing this. Although matching has been discussed since 1962, empirical evidence (especially on school choice) is very recent. Thus, the incentives to develop feasible alternatives, and/or improving features for existing, feasible mechanisms, are fairly new as well. Thus, one should soon be able to observe progress in the overall research field, and to improve the mechanism at UPB.

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Appendix

www.surveymonkey.com/s.aspx?PREVIEW_MODE=DO_NOT_USE_THIS_LINK_FOR_COLLECTION&sm=XnsXCKxJuBls6JyYM6QIW2%2bmJe2dUwgxdLHVv9SC8ql%3d

Kriterien für Studierendenauswahl bei Abschlussarbeiten

**1. Wie in der Email bereits erklärt, geht es um Informationen über die Vorgehensweise der Abschlussarbeitsbetreuer bzw. nach welchem Schema der Lehrstuhl bei der Studierendenauswahl vorgeht.
Es geht nicht um die Fairness oder Qualität Ihrer Auswahl, sondern in erster Linie um die Annahme, dass die Großzahl der teilnehmenden Lehrstühle ähnliche Präferenzen haben könnten.**

Die Antworten werden selbstverständlich vertraulich behandelt, es werden keinerlei Bezüge zur Lehrstuhlinhabern oder Fachgebieten hergestellt.

Ich wäre Ihnen sehr dankbar, wenn Sie die unten genannten Kriterien in die Reihenfolge bringen, mit der Sie bei der Studierendenauswahl angewendet werden (1 als höchste Priorität und 6 als niedrigste Priorität). Hinweis: Die Anordnung der Kriterien aktualisiert sich sofort bei jeder Angabe.

Vielen Dank für Ihre Unterstützung!

- Kurswahl (Themenschwerpunkt im Studium, Belegung von Kursen am eigenen Lehrstuhl)
- Durchschnittsnote (Gesamtleistung des Studierenden)
- Leistung des Studierenden in Fächern des eigenen Lehrstuhls oder in verwandten Modulen (Noten in Kursen in einem bestimmten Themenbereich)
- Lebenslauf (Außeruniversitäre Qualifizierung des Studierenden z.B. Praktika, Arbeitserfahrung,etc.)
- Präsentiertes Interesse des Studenten am Thema (Direkte Gespräche durch den Studierenden, besondere Darlegung der Motivation in einem Schreiben)
- Sonstiges

**2. Falls Sie "Sonstiges" nicht an 6. Stelle eingeordnet haben:
Was ist Ihnen außer den oben genannten Eigenschaften wichtig? Was fällt für Sie unter die Rubrik "Sonstiges" ?**

Fertig

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	1	2	3	4	5	6	Gesamt	Durchschnittliche Rangfolge
Kurswahl (Themenschwerpunkt im Studium, Belegung von Kursen am eigenen Lehrstuhl)	50% 12	12,50% 3	29,17% 7	4,17% 1	4,17% 1	0% 0	24	5,00
Durchschnittsnote (Gesamtleistung des Studierenden)	4,17% 1	12,50% 3	8,33% 2	41,67% 10	25% 6	8,33% 2	24	3,04
Leistung des Studierenden in Fächern des eigenen Lehrstuhls oder in verwandten Modulen (Noten in Kursen in einem bestimmten Themenbereich)	16,67% 4	50% 12	29,17% 7	4,17% 1	0% 0	0% 0	24	4,79
Lebenslauf (Außeruniversitäre Qualifizierung des Studierenden z.B. Praktika, Arbeitserfahrung,etc.)	4,17% 1	0% 0	0% 0	37,50% 9	45,83% 11	12,50% 3	24	2,42
Präsentiertes Interesse des Studenten am Thema (Direkte Gespräche durch den Studierenden, besondere Darlegung der Motivation in einem Schreiben)	16,67% 4	25% 6	25% 6	8,33% 2	20,83% 5	4,17% 1	24	3,96
Sonstiges	8,33% 2	0% 0	8,33% 2	4,17% 1	4,17% 1	75% 18	24	1,79

F2 Exportieren

Falls Sie "Sonstiges" nicht an 6. Stelle eingeordnet haben: Was ist Ihnen außer den oben genannten Eigenschaften wichtig? Was fällt für Sie unter die Rubrik "Sonstiges" ?

Beantwortet: 6 Übersprungen: 18

Beantwortungen (6) | Textanalyse | Meine Kategorien

Kategorisieren als... | Nach Kategorie filtern | Beantwortungen durchsuchen

Anzeigen von 6 Beantwortungen

Leistung des Studierenden in statistischen/mathematischen Fächern
21.01.2014 16:48 Beantwortungen von Befragten anzeigen

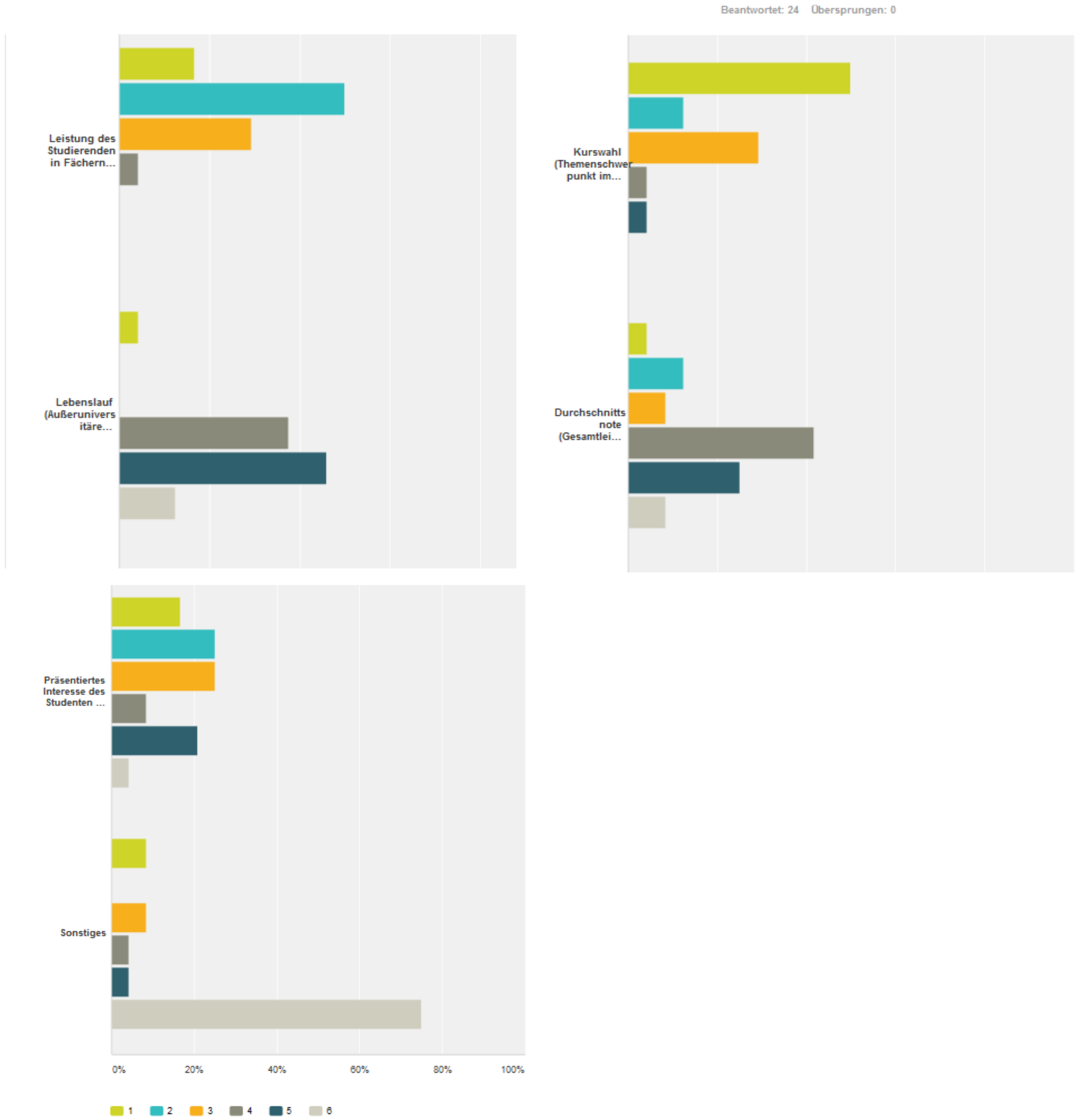
Eindruck darüber, ob der Kandidat in der Lage ist, eine vernünftige Arbeit rechtzeitig abzugeben.
21.01.2014 15:12 Beantwortungen von Befragten anzeigen

vorherige Vorstellung des Studenten / bereits bekannt durch Module oder SHK Tätigkeiten
21.01.2014 10:47 Beantwortungen von Befragten anzeigen

Wenn in der jeweiligen Präferenzstufe weniger Bewerber als Plätze vorhanden sind, werden alle Bewerber angenommen.
21.01.2014 10:38 Beantwortungen von Befragten anzeigen

eigene Idee zur Bearbeitung (Motivation), berufliche Interessen
21.01.2014 10:12 Beantwortungen von Befragten anzeigen

vorherige Kontaktaufnahme zum Lehrstuhl, Bereitschaft und Bemühungen ein eigenes Thema für die Abschlussarbeit zu finden


Beantwortet: 24 Übersprungen: 0
Leistung des Studierenden in Fächern...
Lebenslauf (Außeruniversitäre...
Kurswahl (Themenschwerpunkt im...
Durchschnittsnote (Gesamtlei...
Präsentiertes Interesse des Studenten ...
Sonstiges
0%
20%
40%
60%
80%
100%
1
2
3
4
5
6

Eidesstattliche Erklärung

Hiermit versichere ich, dass ich die vorliegende Arbeit selbstständig angefertigt habe; die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht.

Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

Paderborn, der 25. Februar 2014

[redacted]